Types of analysis: Linear static, linear dynamic and non linear static

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Safety Assessment of Existing Buildings

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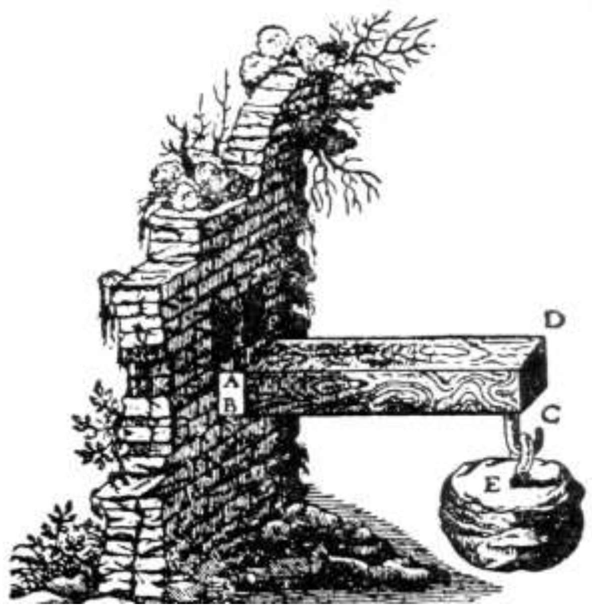
Structural Analysis Methods (Static)

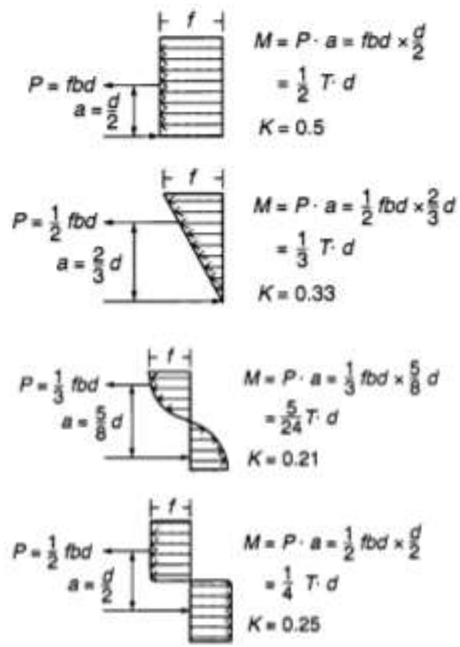
- ❑ Non-linear static analysis
- ❑ Linear static analysis

Simplification

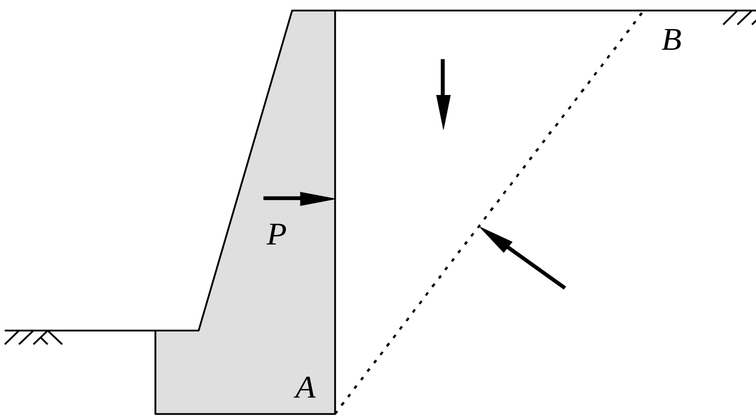
Early Structural Analysis

- "Ut tensio sic vis" or σ / E = ε is the elasticity law established by R. Hooke in 1676.The theory is so extensively used that its limitations and deficiencies are often forgotten. This is in opposition with early forms of *limit analysis*.





Cantilever beam according to Galileo (1638) and evolution of the "hypothesis" for the stress distribution at AB



Retaining wall according to Coulomb (1773)

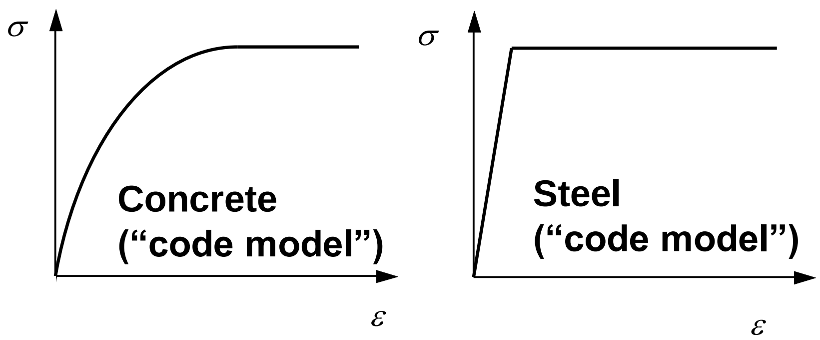
Modern Structural Analysis

- As structural collapse does not generally coincide with the appearance of the first crack or localized early crushing, it seems that the elasticity theory is a step back with respect to limit analysis
- Full nonlinear analysis (the most advanced form of structural analysis) covers the complete loading process, from the initial "stress-free" state, through the weakly nonlinear behavior under service loading, up to the strongly nonlinear behavior leading to collapse
- Interest has been growing since 1970's but it remains a field for selected (few) specialists due to complexity (knowledge) and costs (time) involved
- The possibilities are immense and several commercial software packages include some form of nonlinear behavior, but an incorrect use can be very dangerous

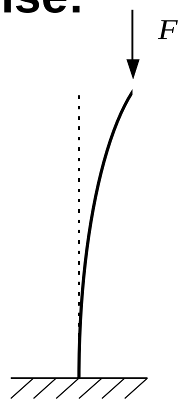
Modern Structural Analysis

- The modern use of nonlinear analysis focuses mostly on these three fields:
  - Complex / stringent safety requirement structures (e.g. nuclear plants, dams, bridges)
  - Virtual laboratory for parametric studies
  - Existing structures (evaluation, repair, rehabilitation)

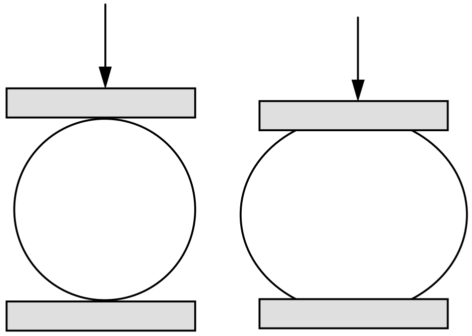
- Three types of non-linearities may arise:



Material (or physical) nonlinearity

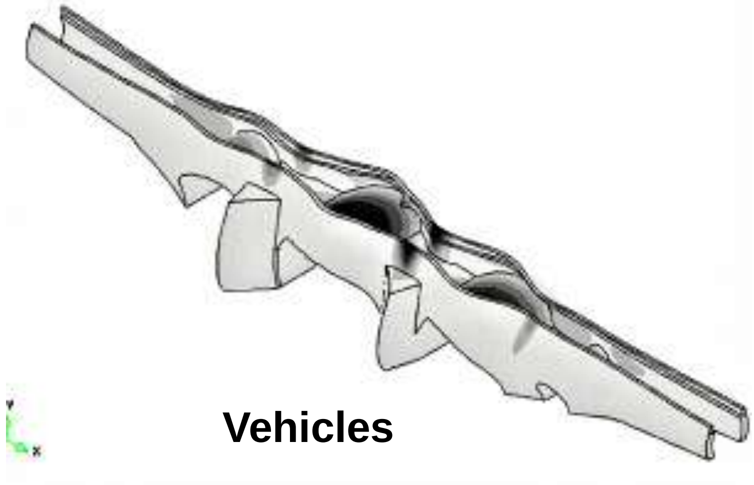


Geometrical nonlinearity

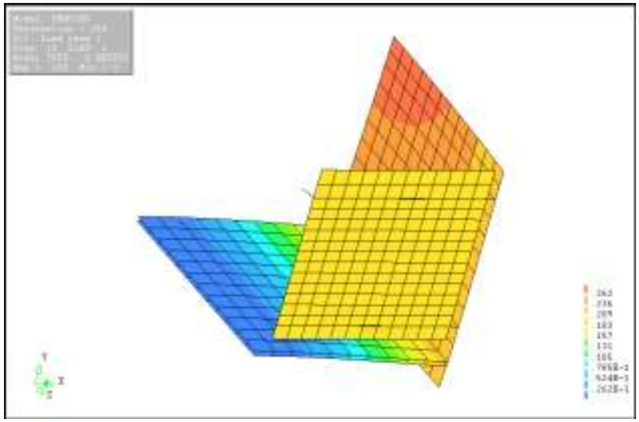


Contact nonlinearity

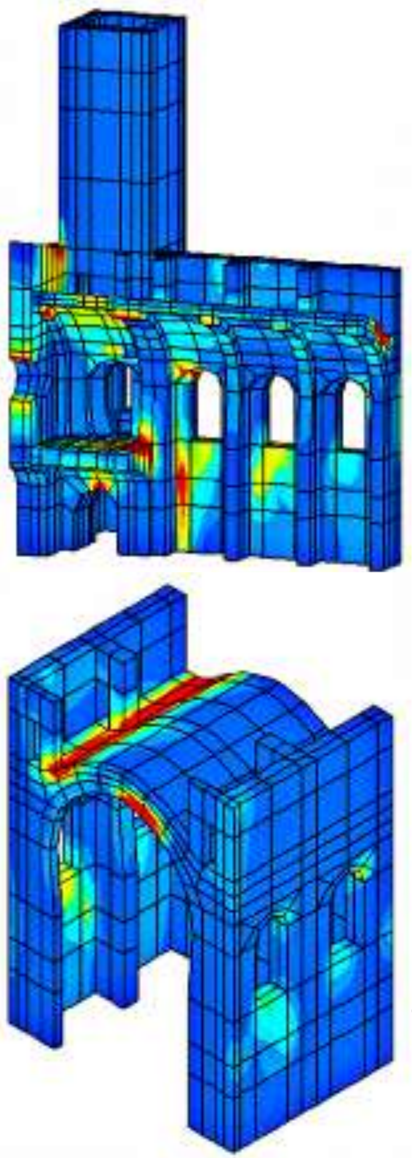
Existing Buildings



Vehicles



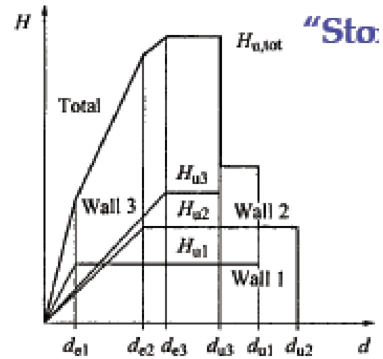
Vertical dead + live load



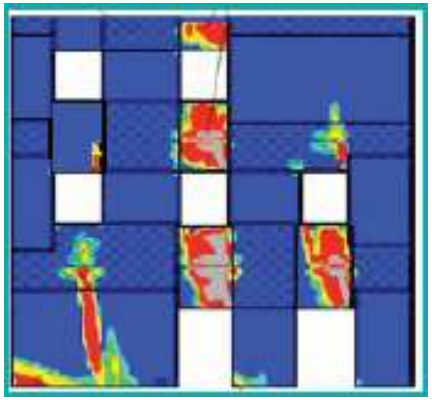
Settlements

Modern Earthquake Design

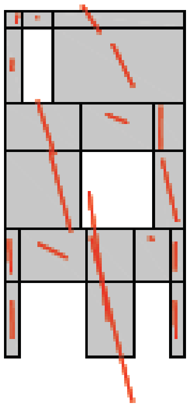
Elastic analysis leads to excessively conservative solutions for unreinforced, confined, and possibly, reinforced masonry

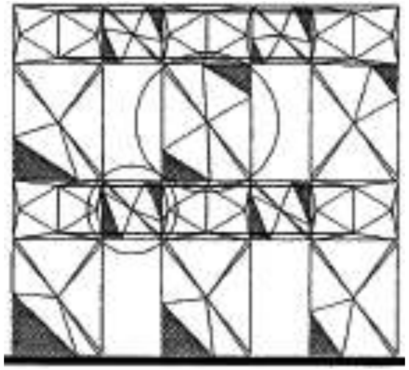


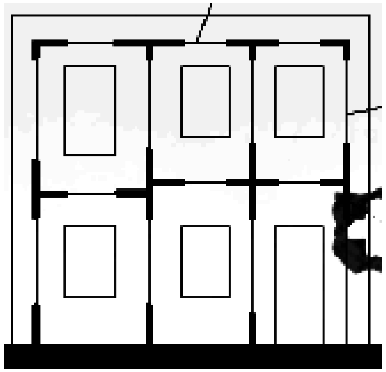
"Storey" model (Por) Tomazevic

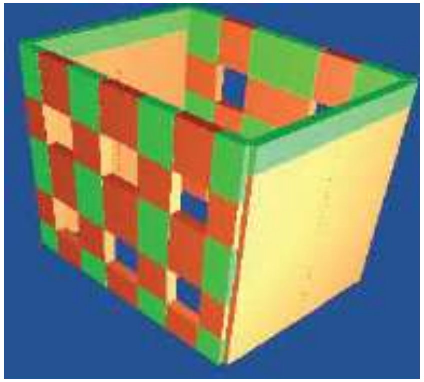


Finite element model (Many authors)





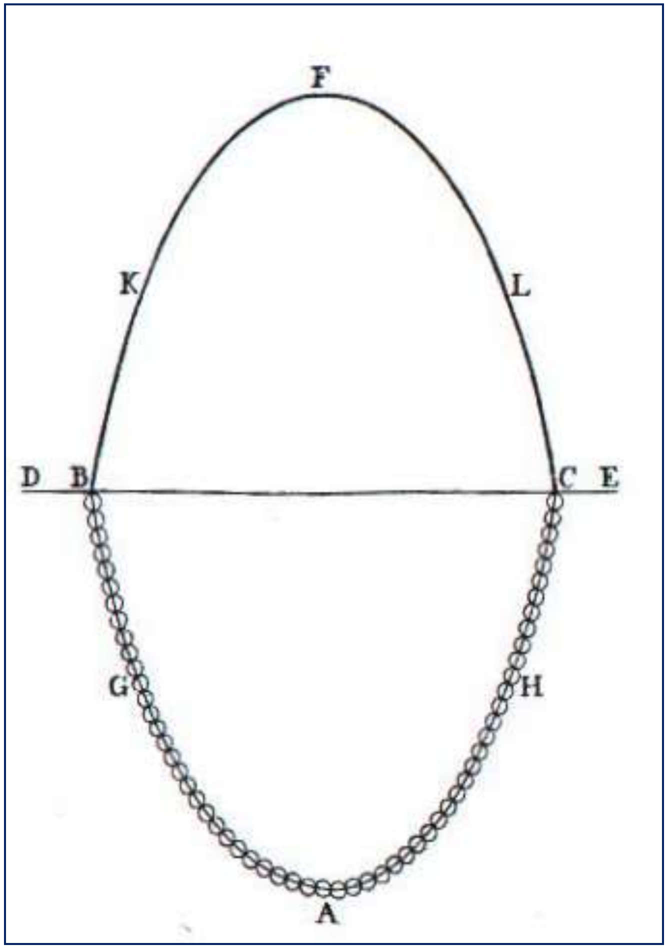




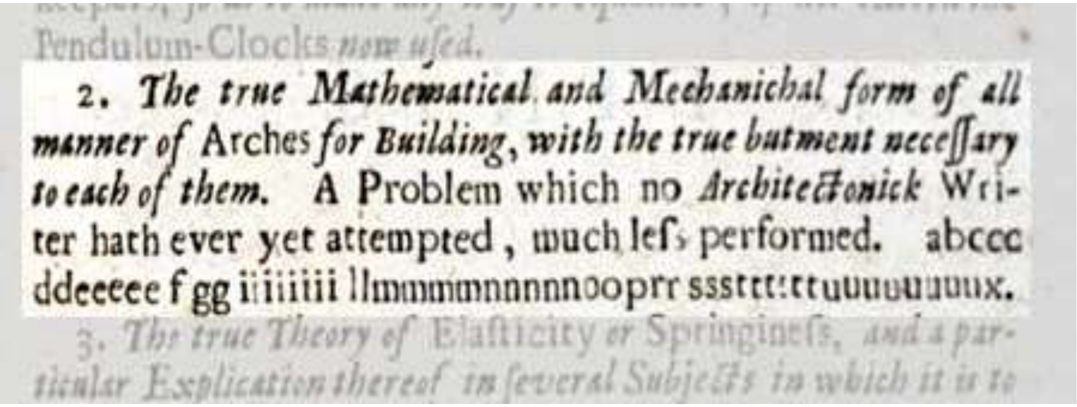
Macro-models (Braga, Liberatore, D'Asdia, Magenes, Lagomarsino, etc.)

Example of Analysis of an Arch

Robert Hooke (1635-1703)- Principle of the inverted catenary



In 1675 Hooke provided the solution for he equilibrium of an arch by means of an anagram included in the book *"A description of Helioscopes and some other Instruments",* which was only deciphered after his death in 1703.



Pendulum-Clocks now ufed.

2. *The true Mathematical and Mechanichal form of all manner of* Arches *for Building, with the true butment neceffary to each of them.* A Problem which no *Architectonick* Writer hath ever yet attempted, much lefs performed. abccc ddeeeee f gg iiiiiiii llmmmmnnnnnooprr ssstttttuuuuuuuux.

3. *The true Theory of* Elafticity *or* Springinefs, *and a particular Explication thereof in feveral Subjects in which it is to*

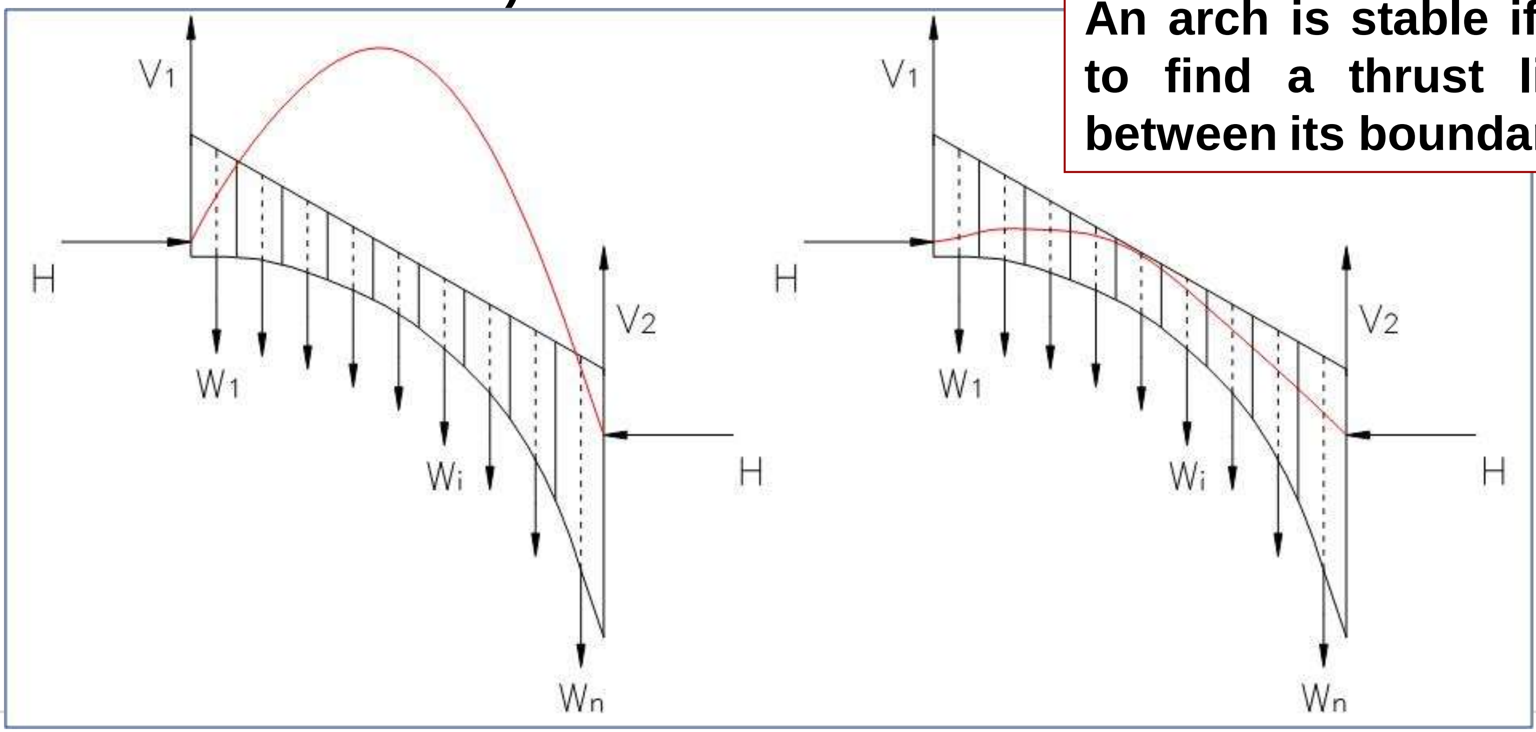
***The solution*: "Ut pendet continuum flexile, sic stabit contiguum rigidum inversum" – as hangs the flexible line, so but inverted will stand the rigid arch.**

Graphic Statics

The arch is first decomposed in a series of real or fictitious voussoirs separated by a series of planes (the planes do not need to be parallel)

The thrust line is defined as the geometrical locus of the points of application of the sectional forces (the resulting forces over each plane between voussoirs) across the arch

An arch is stable if it is possible to find a thrust line contained between its boundaries



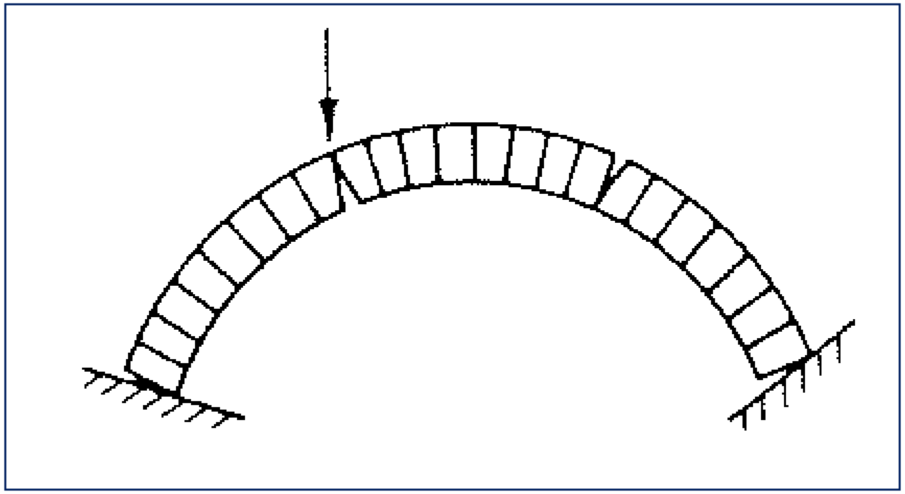
Kinematic Analysis

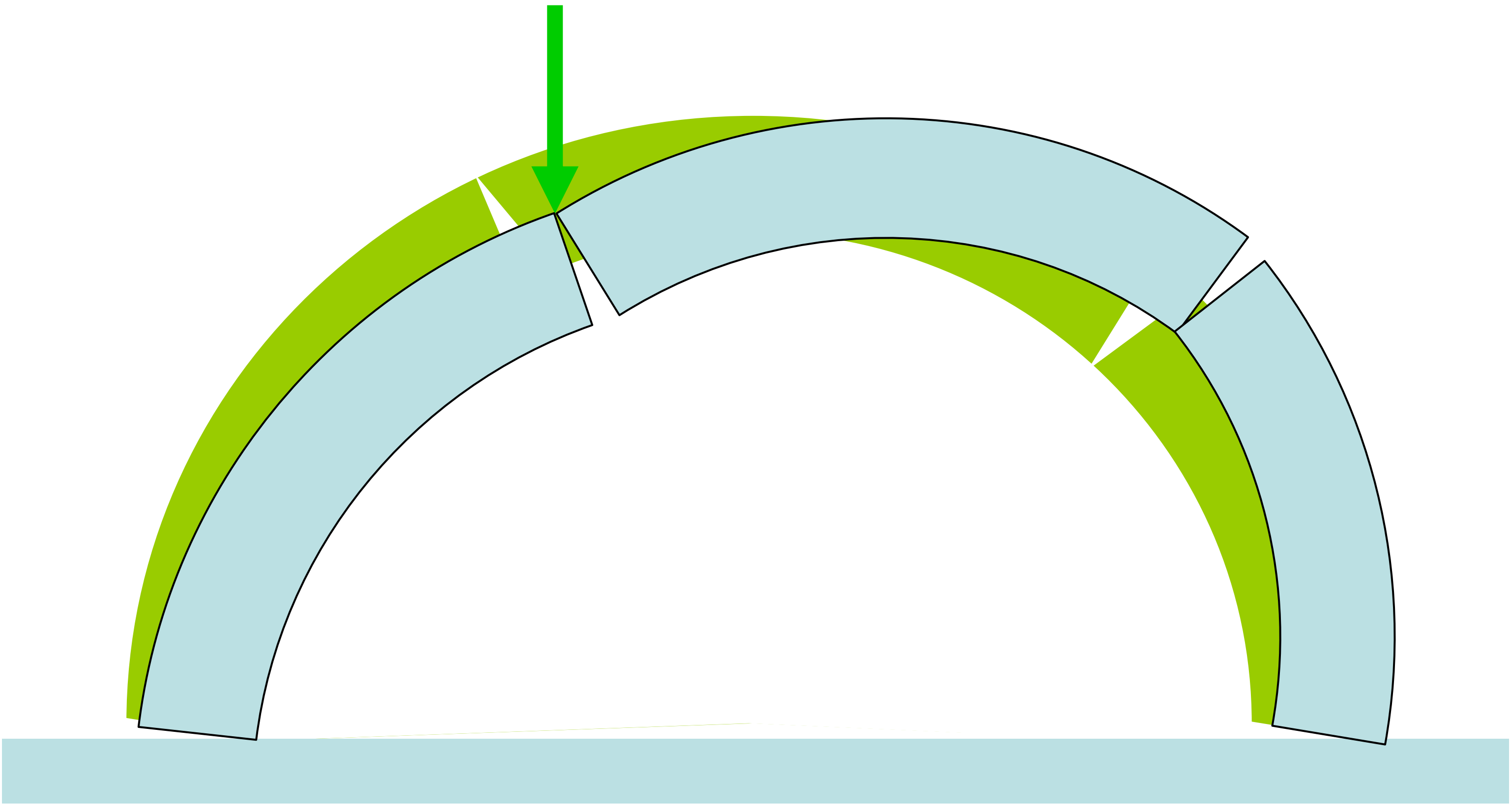
Charles-Agustin COULOMB (1736-1806) proposed in 1773 the first general and accurate theory on the stability of masonry arches

The basic assumptions are:

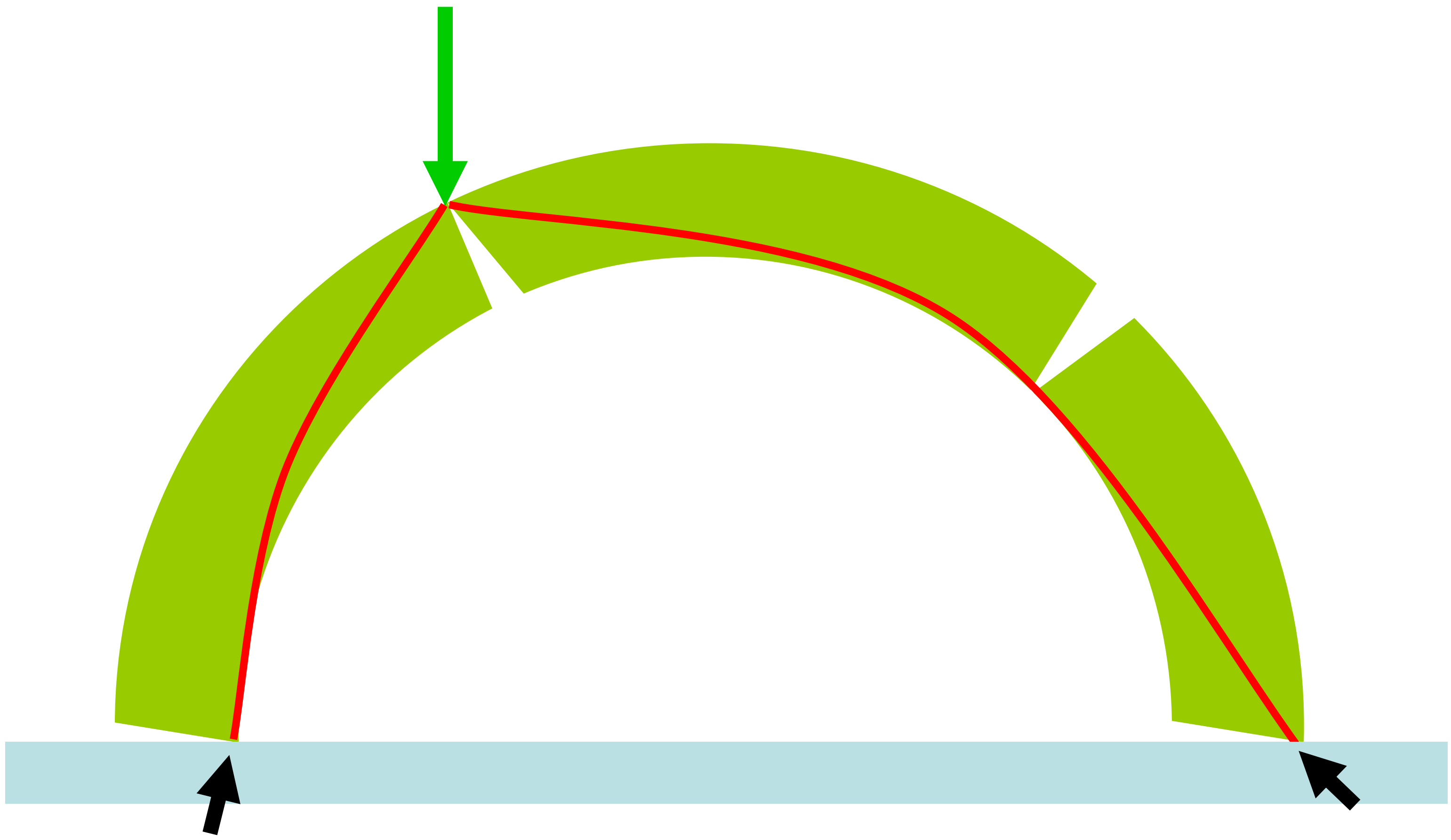
(1) Sliding between voussoirs is unlikely due to the existing frictional forces

(2) Collapse will be caused by the rotation between parts due to the appearance of a number of hinges. The location of the hinges is a priori unknown but can be determined by the method of "maxima and minima"

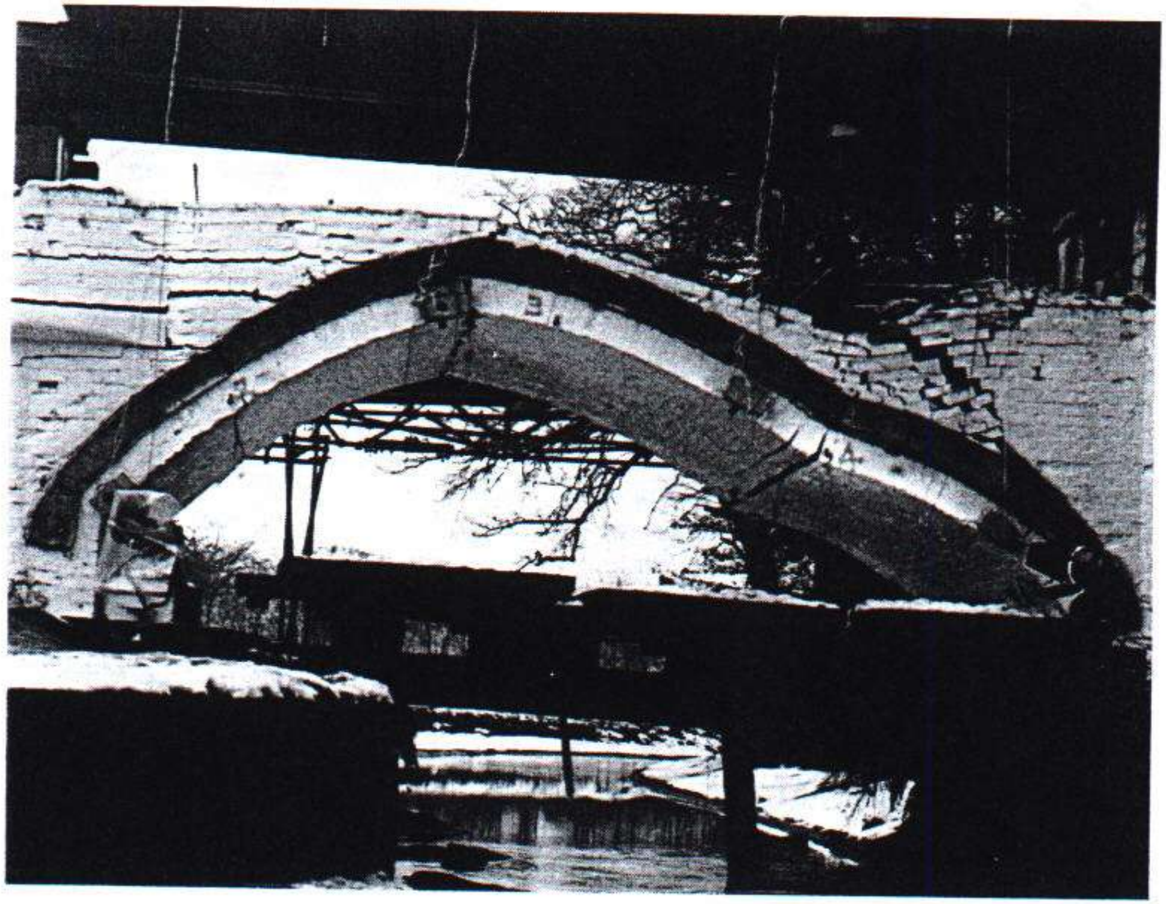




Kinematics of 4-hinge collapse



Correspondence with THRUST LINE theory: a hinge will develop each time the equilibrium line becomes tangent to an alternate boundary. In this condition (failure), the thrust line is determined and unique.



Collapse of an arch brought experimentally to failure

Static Analysis Methods (I)

- **Linear Elastic Analysis**

  elastic properties + maximum admissible stress

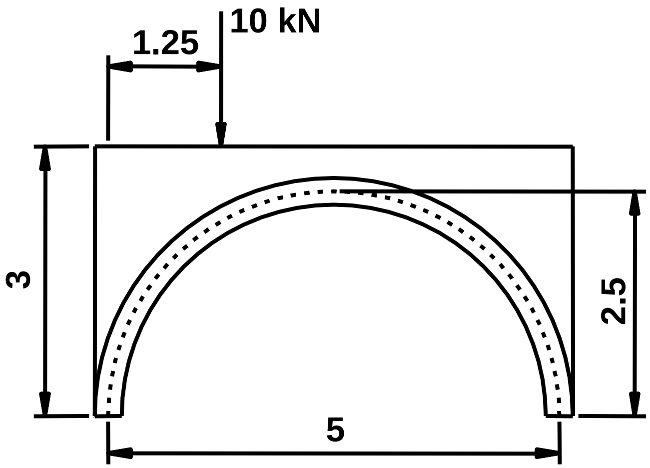
- **Kinematic Collapse Mechanism Analysis**

  inelastic properties = friction angle + tensile and compressive strengths

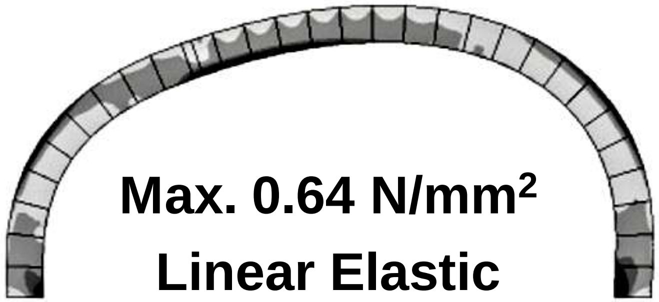
- **Static Thrust Line Analysis**

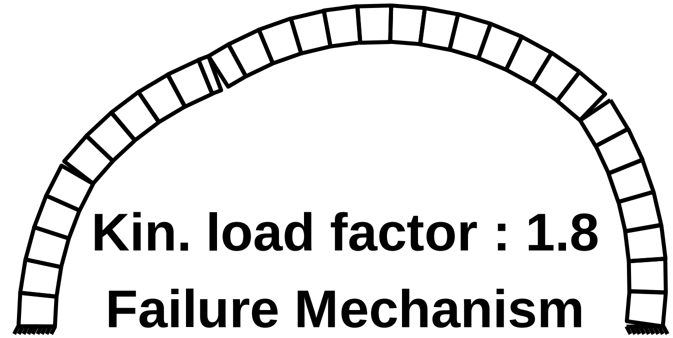
- **Non-linear Analysis (Physical and Combined)**

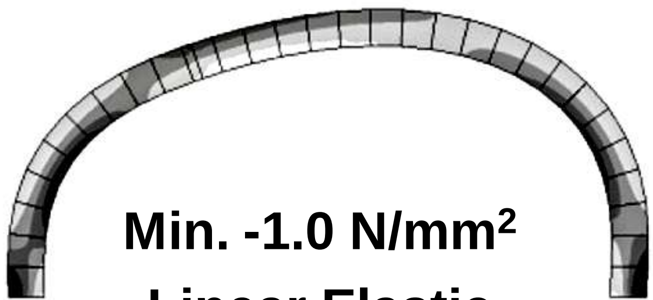
  FULL inelastic properties (ft = 0 and ft ≈ 0) + elastic properties

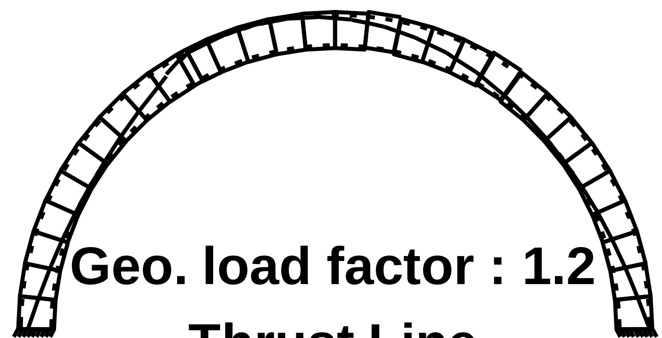


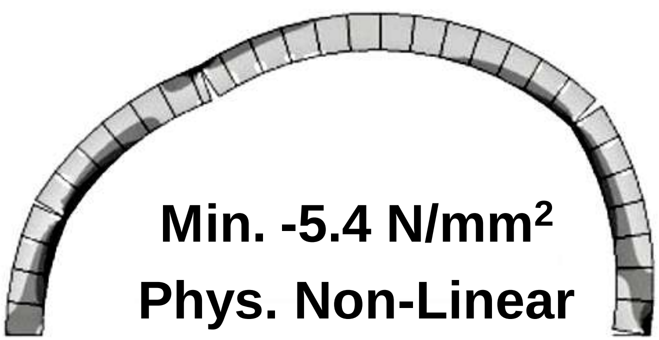
Static Analysis Methods (II)

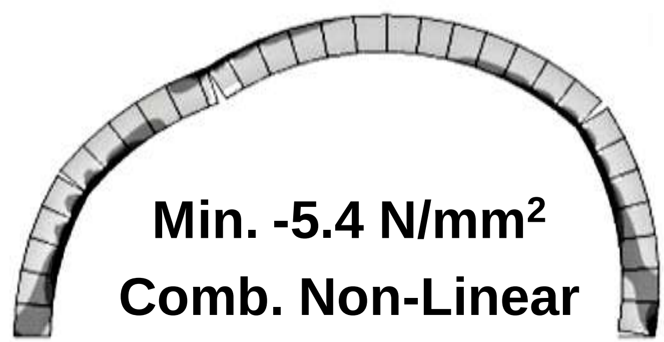




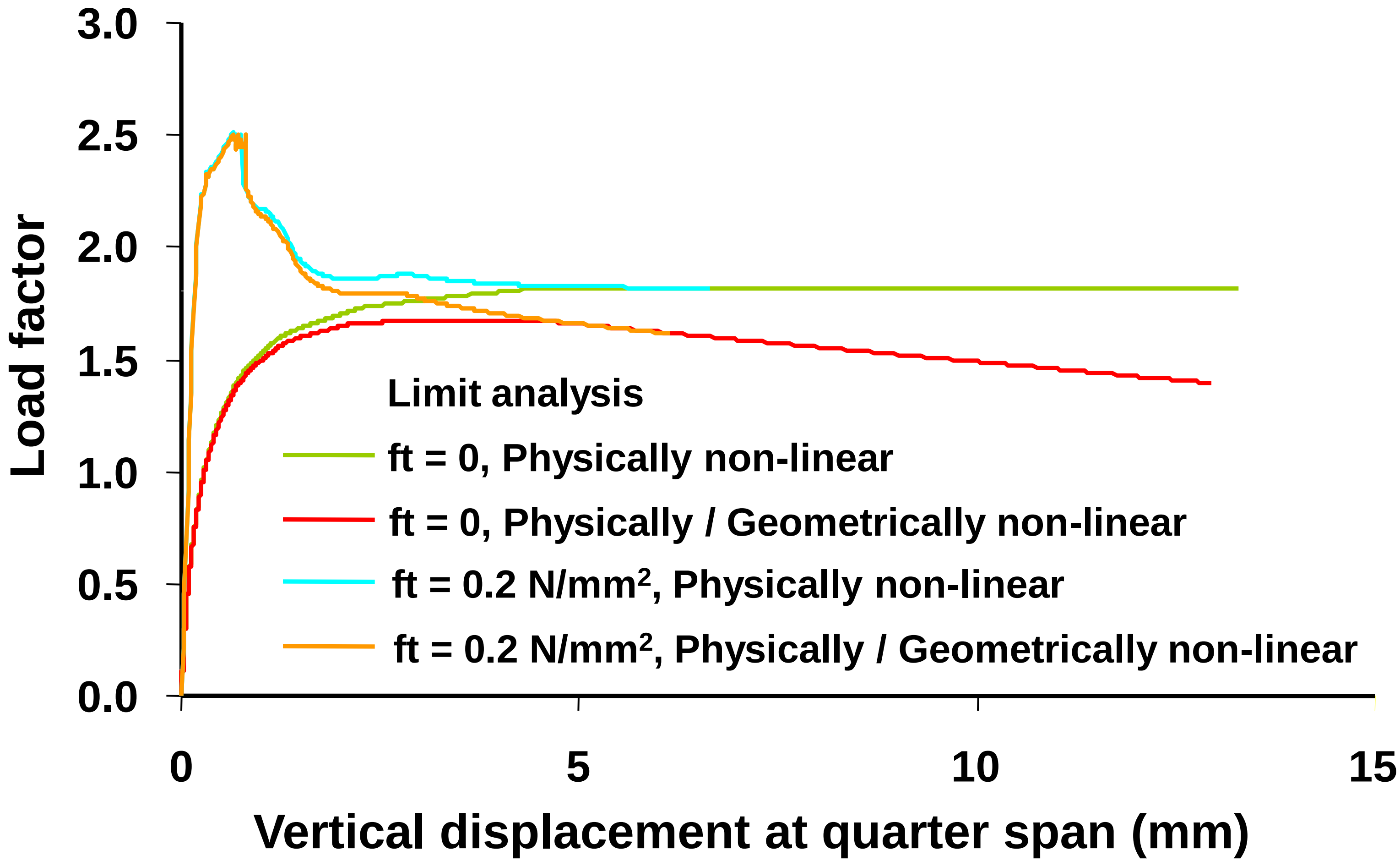








Static Analysis Methods (III)

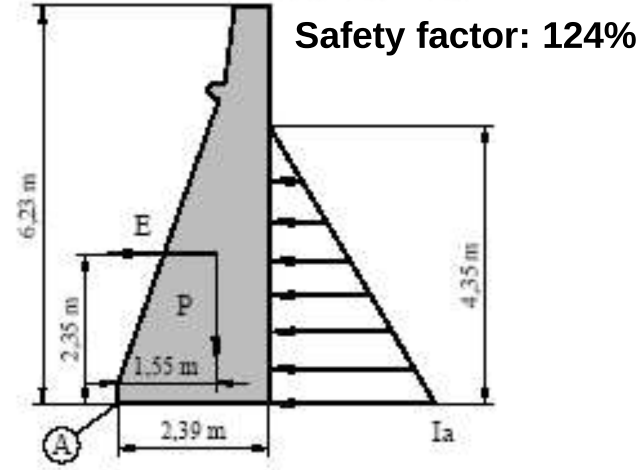


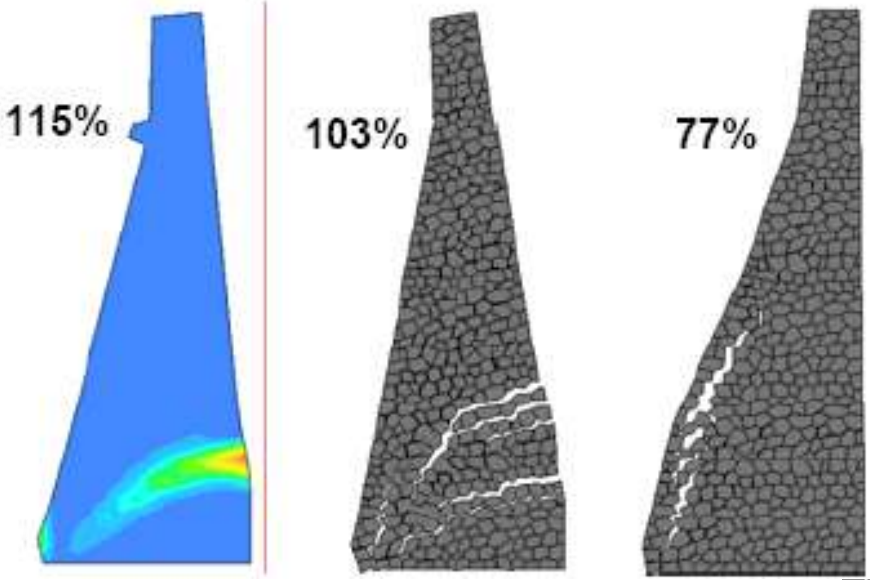
Static Analysis Methods (III)

Approach/Analysis type	Semi-circular arch
Allowable stresses (f_{ta}=0.2 N/mm^2)	0.31
Kinematic limit analysis	1.8
Geometric safety factor	1.2
f_t = 0, Physically non-linear	1.8
f_t = 0, Physically and geometrically non-linear	1.7
f_t = 0.2 N/mm^2, Physically non-linear	2.5
f_t = 0.2 N/mm^2, Phys. and geom. non-linear	2.5

- The "safety factors" of a linear elastic analysis and a static limit analysis cannot be compared with the remaining safety factors.
- Physically non-linear analysis and kinematic limit analysis yield the same failure mechanisms and safety factors?
- The consideration of a non-zero, yet low and degrading, tensile strength increased the safety factors considerably. The post-peak is a key issue.
- Different methods of analysis lead to different safety factors and different completeness of results.

More on Static Analysis Methods…





Structural Analysis Methods: “Dynamic”

- Non-linear time history analysis
- Non-linear static analysis

- Linear elastic time history analysis
- Modal superposition
- Linear static analysis

Simplification

Push-Over Analysis (I)

- In the recent years new methods of seismic assessment and design have been developed, particularly with respect to push-over analysis

- Two methods of analysis can be distinguished:

  - Traditional force method, combined with control of performance requirements based on deformation

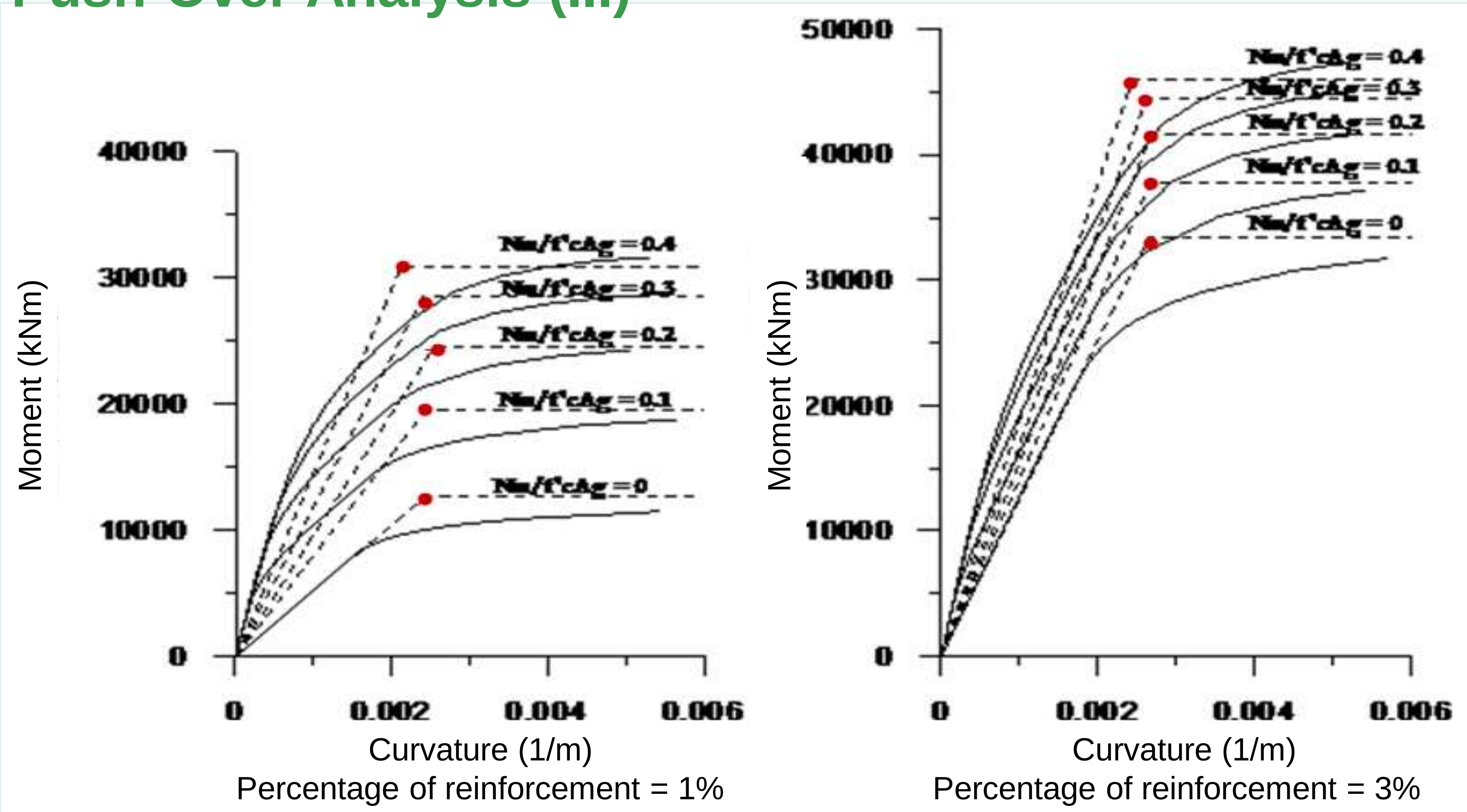
  - Displacement based method, in which the analysis starts by defining a target displacement (measuring the structural response).

Push-Over Analysis (II)

Traditional Force Method

- The dimensions of the structural members are considered
- The stiffness of the members is also considered (the codes might consider elastic stiffness or 30 to 50% of the elastic stiffness)
- Periods are based on stiffness (Note: The design forces can be reduced about 30 to 50% if the stiffness is reduced to the half)
- Forces are distributed in the elements according to the stiffness

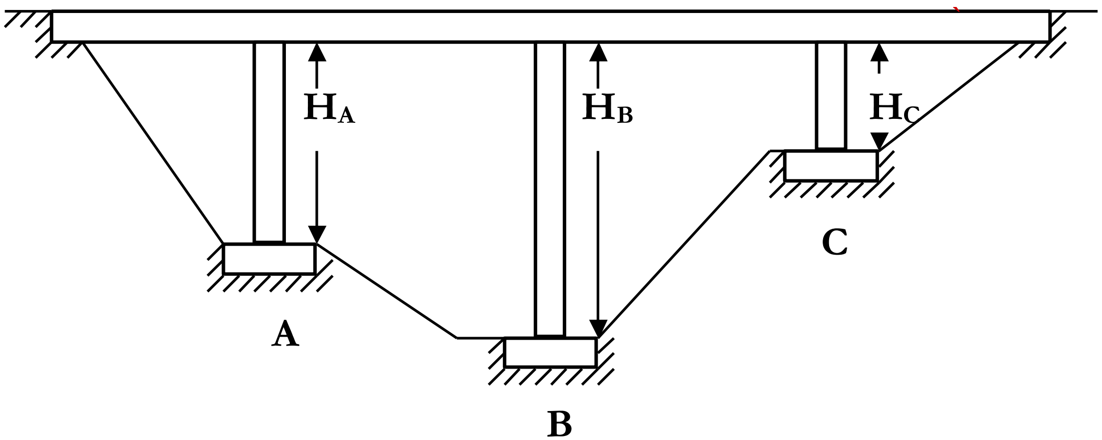
Push-Over Analysis (III)

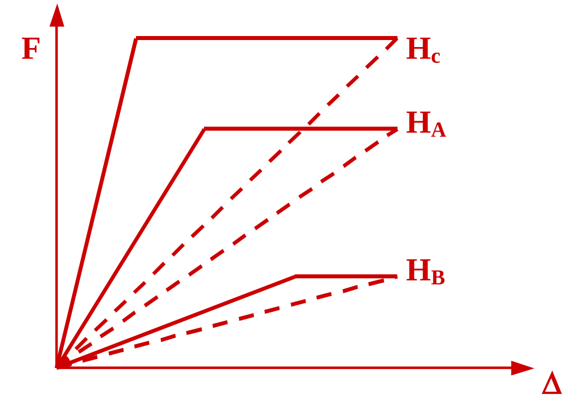


Moment-curvature curves for circular columns (D=2 m, f_c=35 MPa, f_y=450 MPa)

Stiffness and strength are correlated!!

Push-Over Analysis (IV)





	Force Based Design	Displacement Based Design
Stiffness:	proportional to $1/H^3$	proportional to $1/H$
Shear:	proportional to $1/H^3$	proportional to $1/H$
Moment:	proportional to $1/H^2$	equal
Reinforcement:	proportional to $1/H^2$	equal
Ductility:	equal (!)	proportional to $1/H^2$

Masonry Structures With Box Behavior

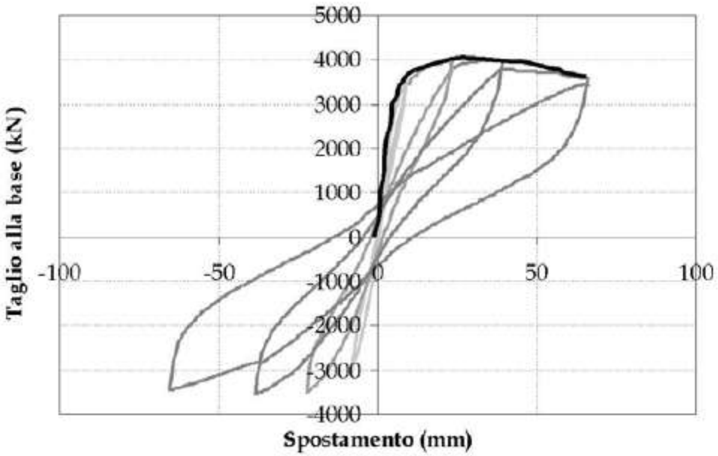
המועצה לשימור
אתרי מורשת בישראל

Universidade do Minho

Recent test results: Rigid diaphragm

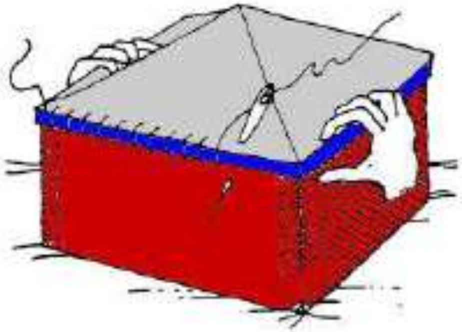
- Worst case scenario: Embedded ring beam + Unfilled vertical joints
- Moderate damage up to 100% of the design earthquake in Lisbon
- Ductile failure for 250% of the design earthquake in Lisbon

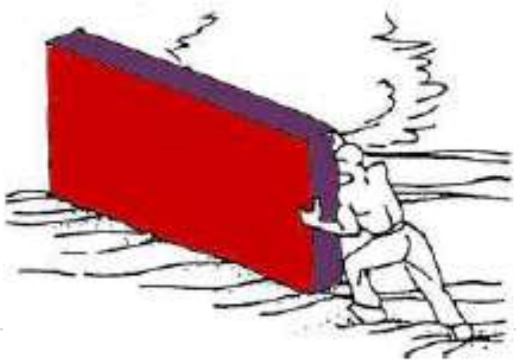
- Experimental results show that URM possesses considerable capacity for inelastic deformations, and then the application of nonlinear analysis is obvious



- Seismic pushover analysis simulates the evolution of the condition of structures during earthquakes, through application of incremental horizontal forces until collapse

- Assumptions of box behaviour and in-plane response are considered

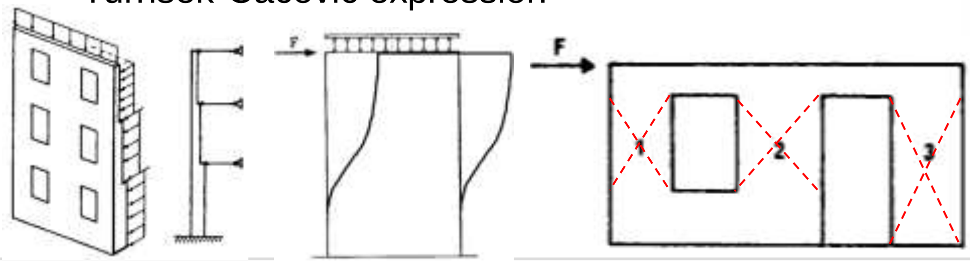


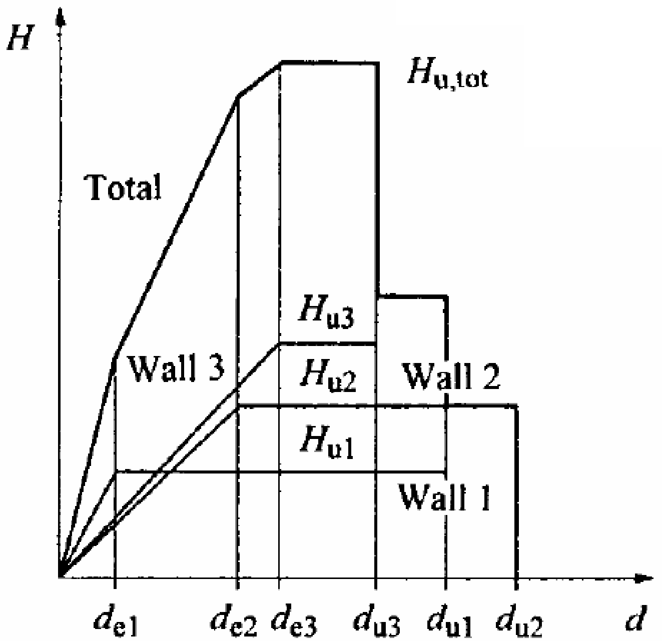


"POR" Storey Mechanism

- Developed in former Yugoslavia and Italy as a reaction of the Skopje earthquake in 1963, and implemented in the region Friuli-Venezia Giulia after the Friuli earthquake in 1976 (DT2, 1977)

- **The following hypothesis are considered:**
  - Thickness of the wall is constant in each level
  - Slabs are rigid in-plane diaphragms
  - Ends of the piers do not rotate, but only suffer translation
  - Behavior of the piers is elastic-perfectly plastic, with a predefined ductility
  - Elastic stiffness of each panel remains constant
  - Panels collapse by diagonal shear according the Turnsek-Cacovic expression

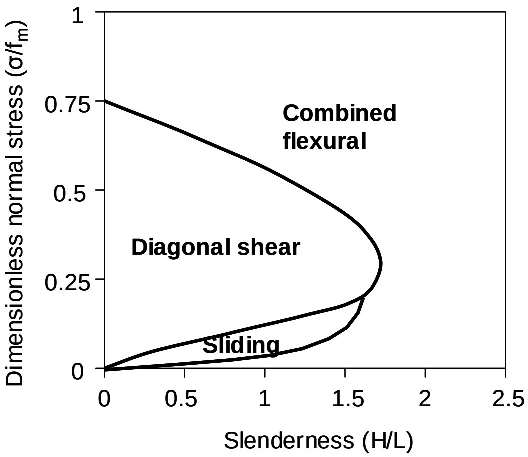


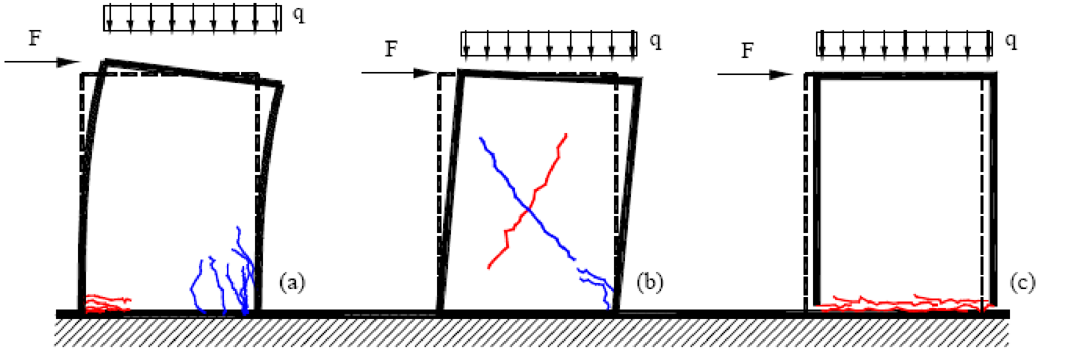


Tomaževič, Braga & Dolce

Additional Macro-Mechanisms

Since the 1980s, observation of damage in masonry buildings subjected to significant vertical load due to use of slabs, and constituted by slender piers, introduces a new trend of research on the combined flexural mechanism



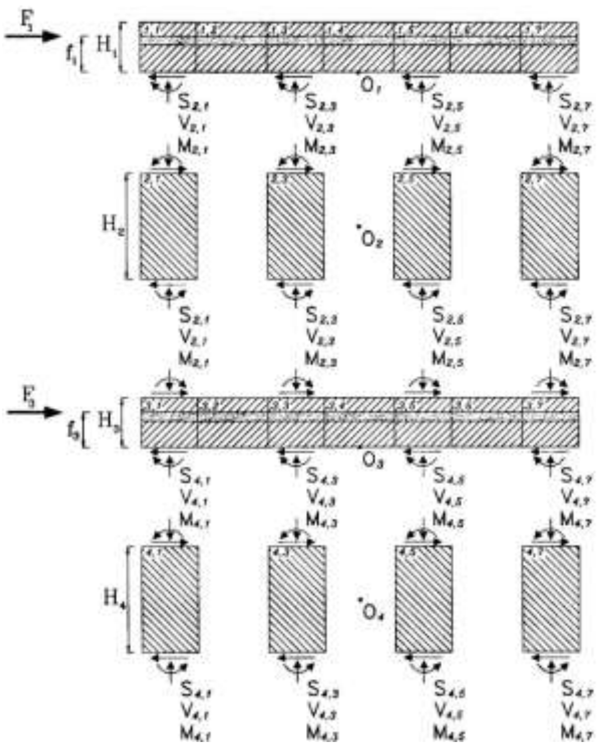


Comb. flexural **Diagonal shear** **Sliding shear**

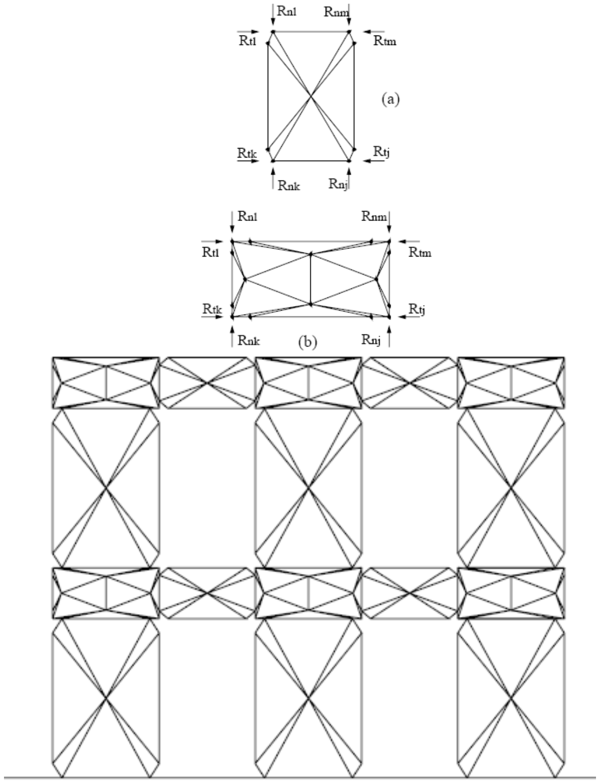
Mixed

Early Improvements in Italy

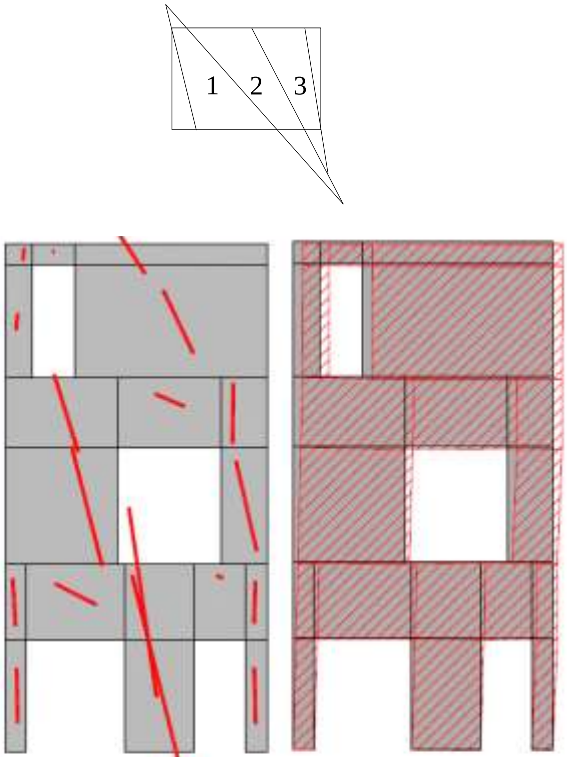
- Initially these methods only had an impact in the scientific community and the POR persisted as the method most used by Italian designers



RAN
(Raithel & Augenti)



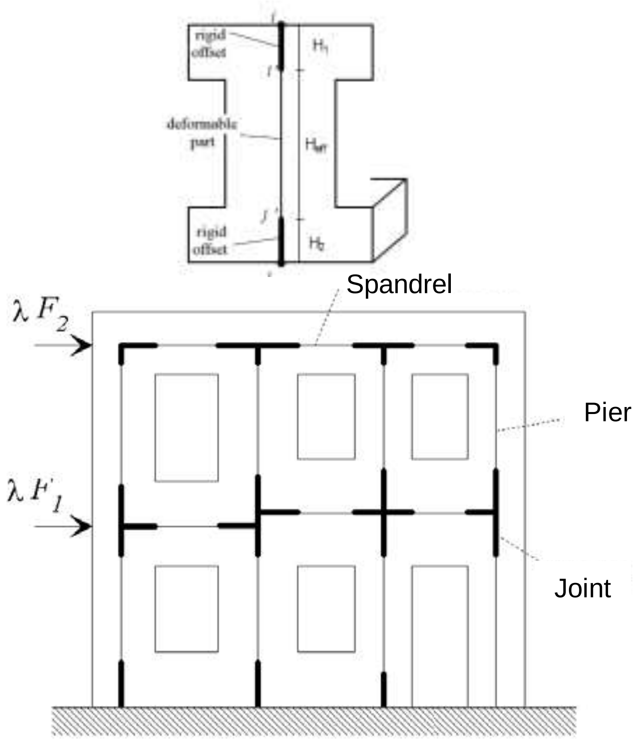
PEFV
(D'Asdia & Viskovic)



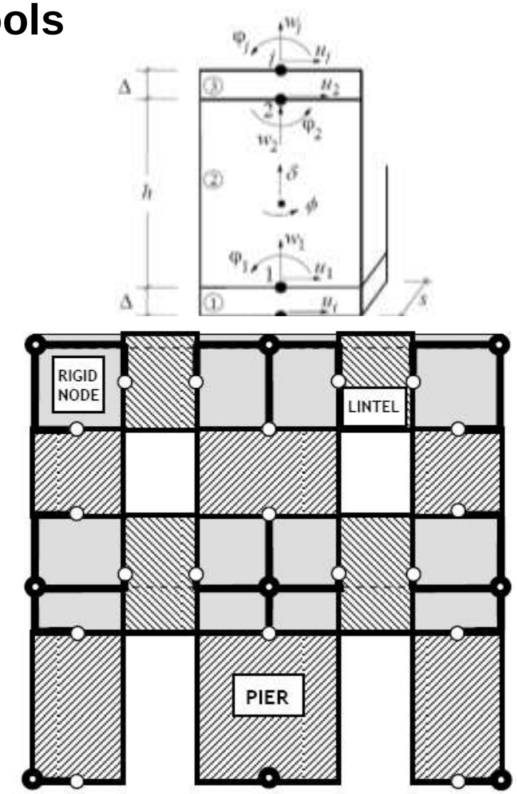
MAS3D
(Braga et al.)

A Generation of Design Methods in Italy

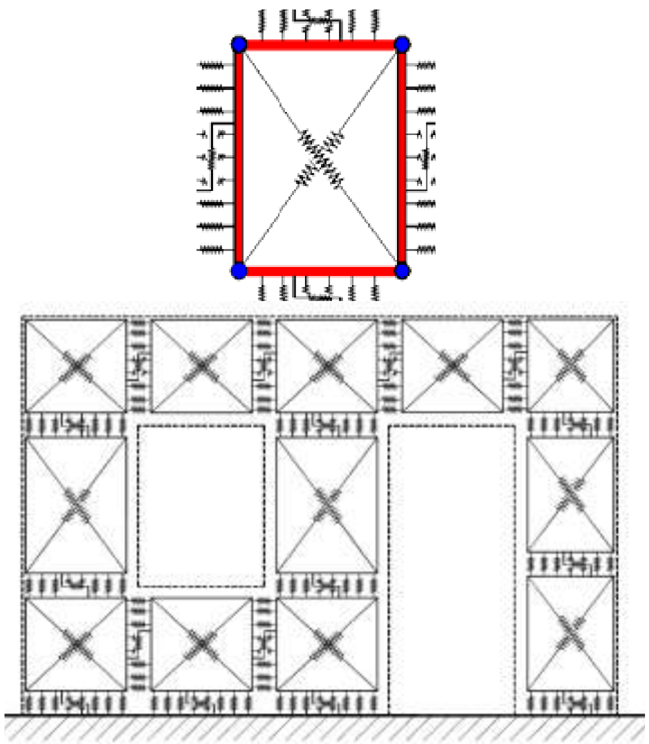
- **As a consequence of the 2002 Molise Earthquake the new Italian code OPCM 3274/2003(3431/2005) was introduced, and macro-elements methods emerge as modern and practical tools**



SAM
(Magenes et al.)

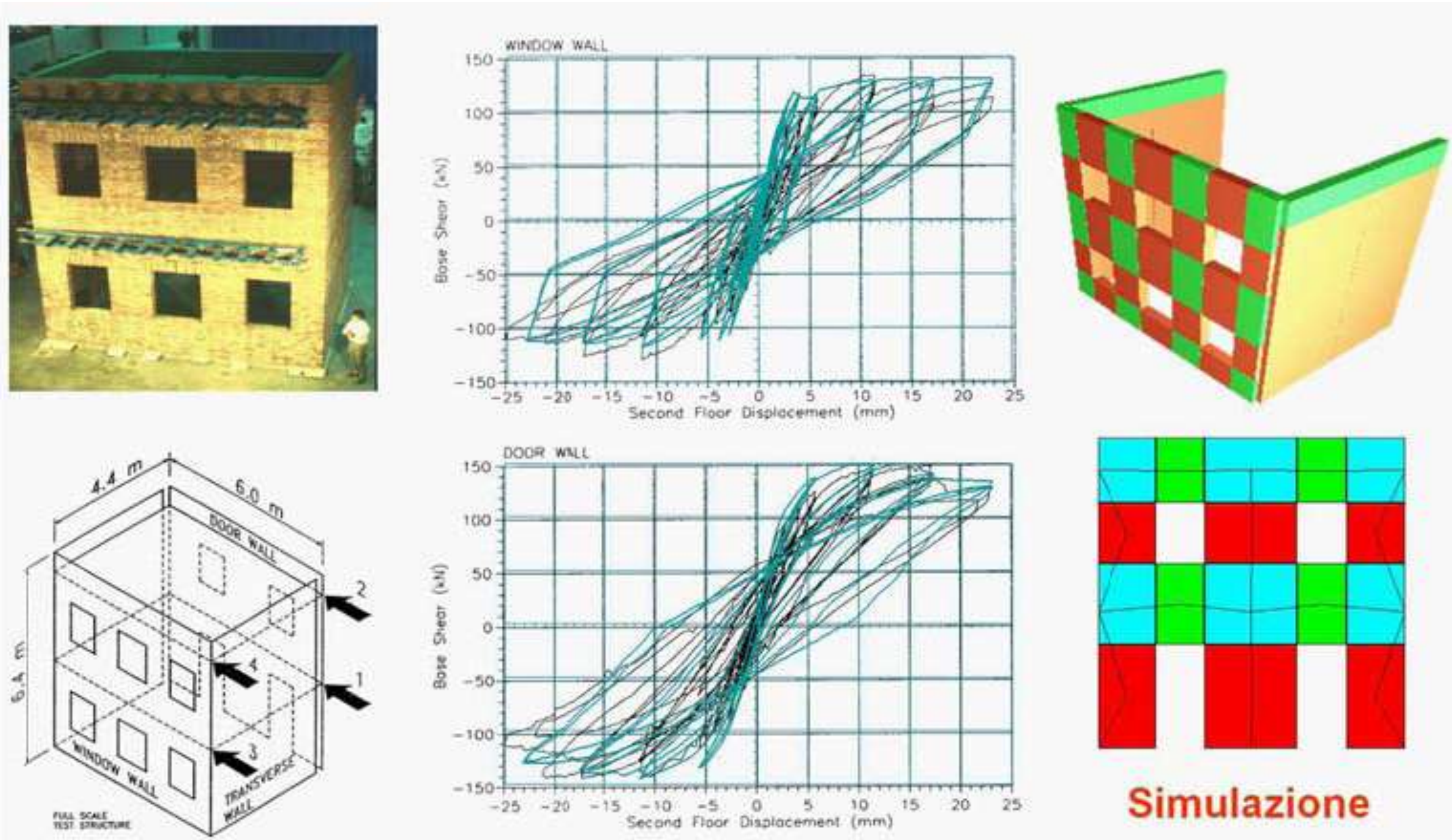


3Muri
(Lagomarsino et al.)



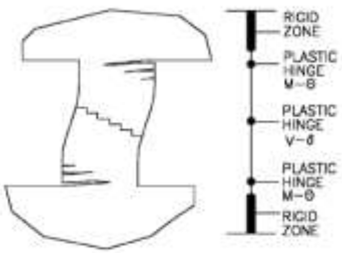
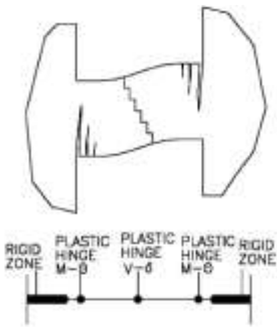
3DMacro
(Caliò et al.)

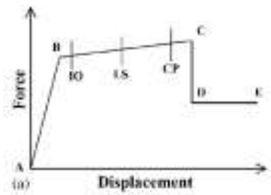
Validation Example

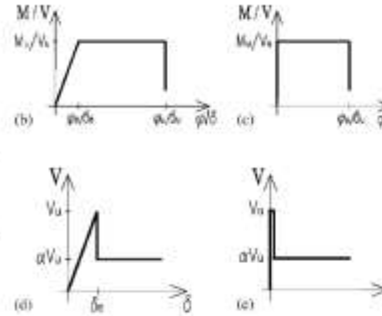


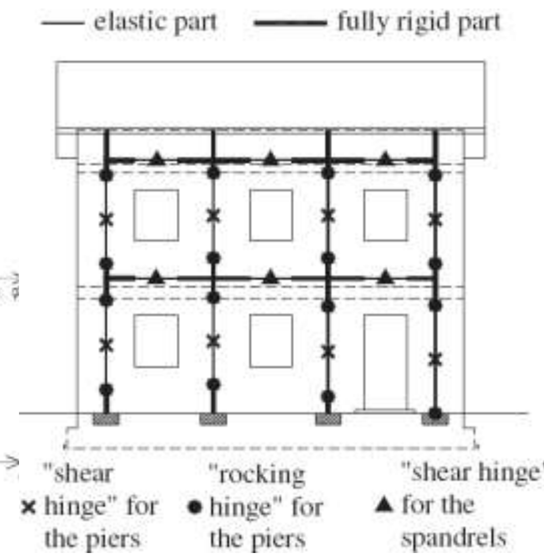
Or even use SAP 2000

One-dimensional macro-element

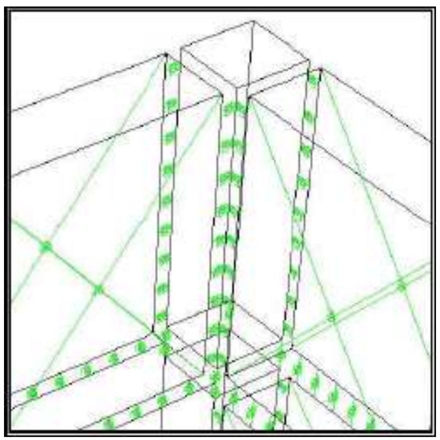
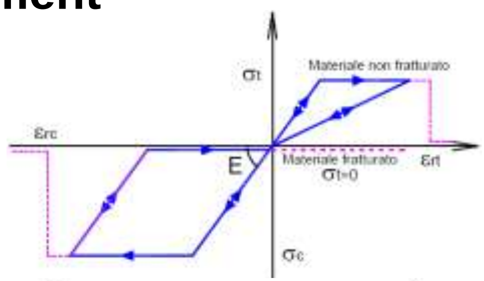



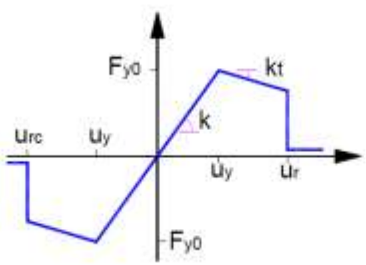


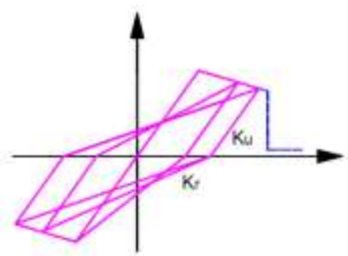


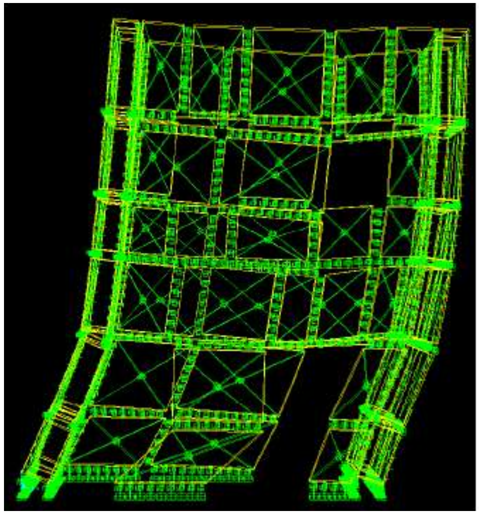


Bi-dimensional macro-element







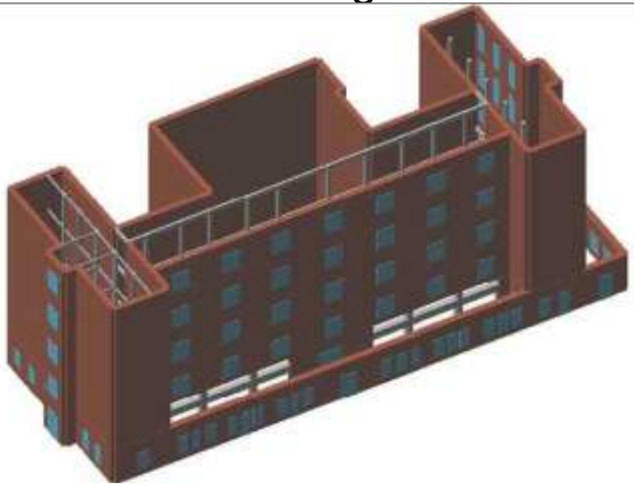
Commercial Software (I)

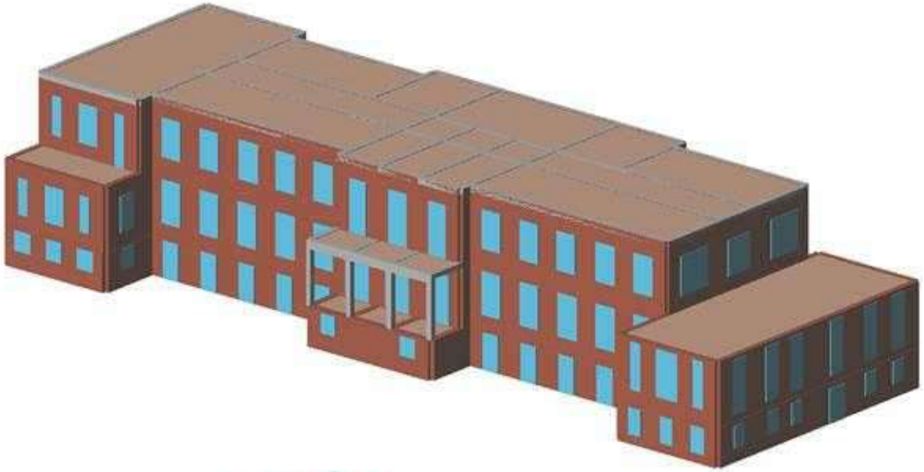
There is commercial software available for structural UR masonry, particularly in Italy. Benchmarking was made in two publications: Azores 1998, Eds. C. Sousa Oliveira et al., (2008) and Marques, R., Lourenço, P.B., Possibilities and comparison of structural component models for the seismic assessment of masonry buildings, Computers and Structures, 89 (21-22), p. 2079-2091 (2011)

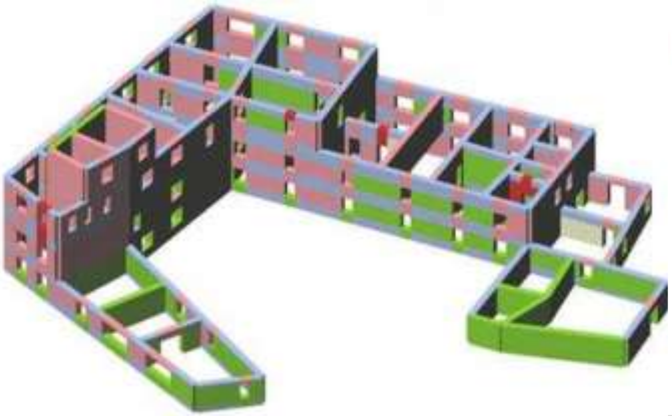
Program	Country	Code	Approach	Web adress
AEDES	Italy	Italian	FEM and SCM	www.aedes.it
CMT+L	Spain	Eurocode	FEM	www.arktec.com/cmtl.htm
FEDRA	Norway	Eurocode	FEM	www.runet-software.com/FEDRA.htm
WIN-Statik MurDim+	Sweden	?	?	www.strusoft.com
Por 2000	Italy	Italian	SCM	www.newsoft-eng.it/Por2000.htm
TQS CAD/Alvest	Brazil	Brazilian	?	www.tqs.com.br/v13/alvest.htm
Tricalc.13	Spain	Eurocode	FEM	www.arktec.com/new_t13.htm
Tricalc.17	Spain	Spanish	FEM	www.arktec.com/new_t17.htm
WinMason	USA	USA	Storey Mech.	www.archonengineering.com/winmason.html
3DMacro	Italy	Italian	SCM	http://www.3dmacro.it/
3Muri	Italy	Italian	SCM	www.stadata.com
ANDILWall	Italy	Italian	SCM	www.crsoft.it/andilwall
MURATS	Italy	Italian	Storey Mech.	www.softwareparadiso.it/murats.htm
Sismur	Italy	Italian	Storey Mech.	www.franiac.it/sismur.html
TRAVILOG	Italy	Italian	Storey Mech.	www.logical.it/software_travilog.aspx
Tecnobit	Italy	Italian	Storey Mech.	www.tecnobit.info/products/murature.php
CDMaWin	Italy	Italian	FEM and SCM	www.stsweb.net/STSWeb/ITA/homepage.htm

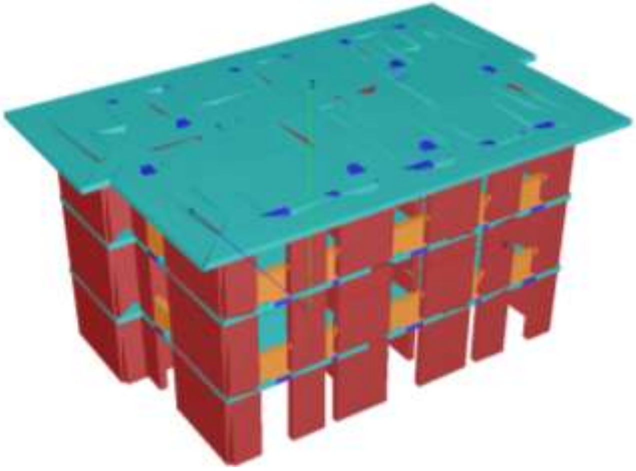
Commercial Software (II)

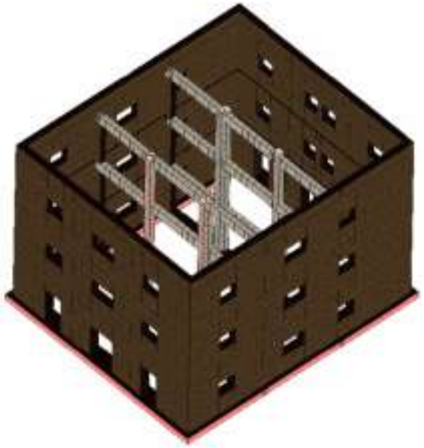
- Efficient and high level modeling





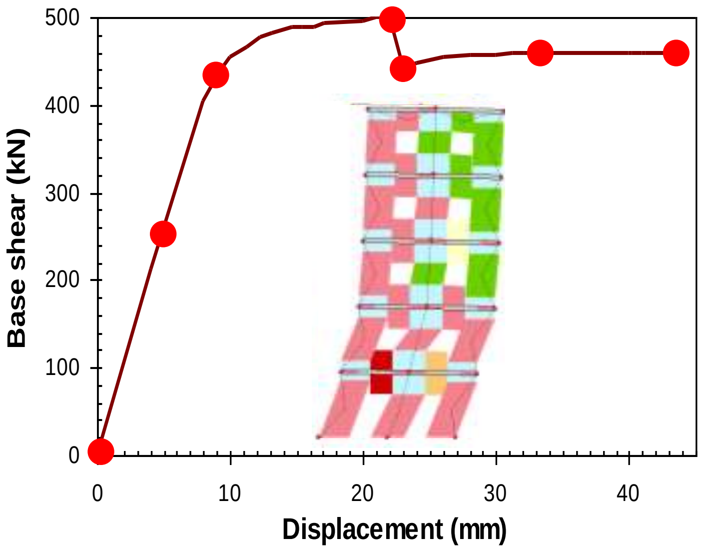


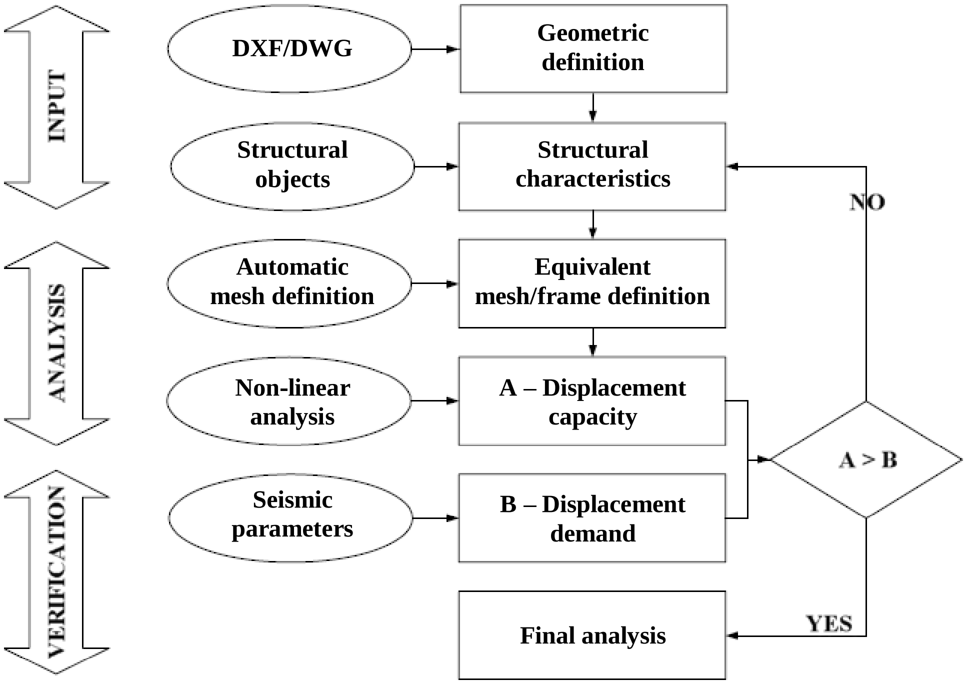




Displacement Based Design

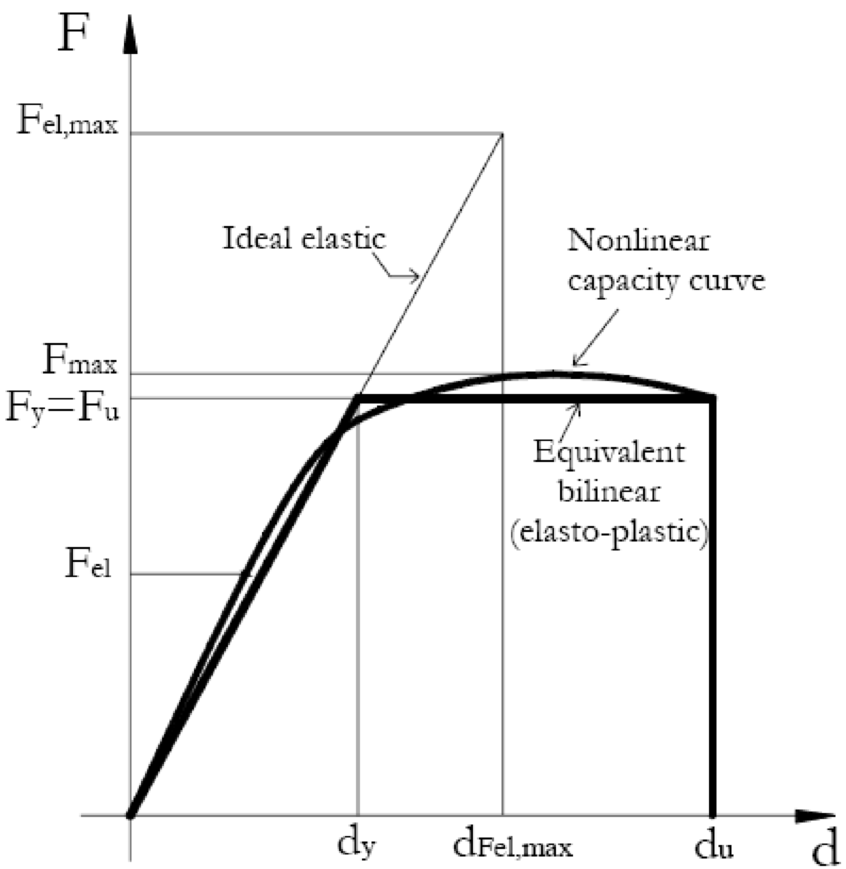
- Recent methods implement capacity/displacement-based seismic design, by evaluating the evolution of damage and displacement
- If the damage evolution can be used as a measure of seismic performance, the confrontation between displacement capacity and displacement demand is the rule for safety verification





Energy Dissipation Capacity (I)

In a force based method, the non-linear reserve capacity must be considered



For unreinforced masonry buildings with 2 or more storeys:

EC8:

q = 1.5-2.5 (recommended 1.5)

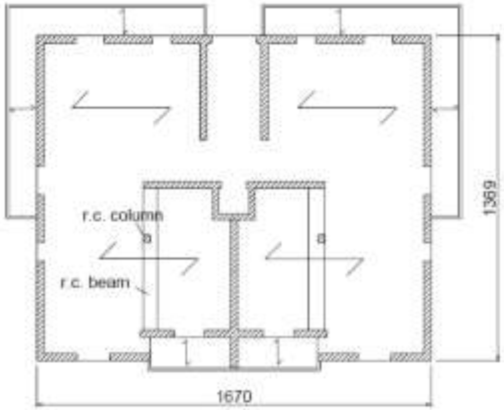
OPCM 3431:

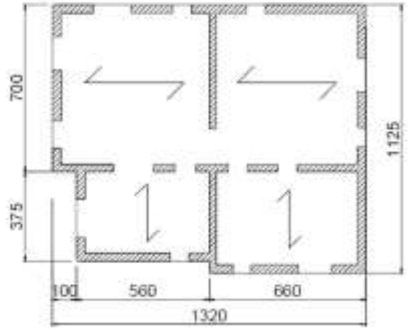
α_u / α_1 (OSR) = 1.8

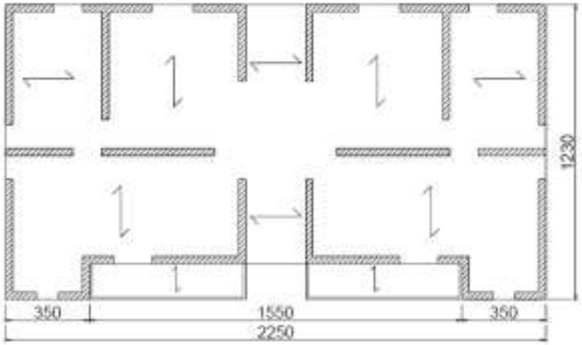
$q = q_0 \times OSR = 3.6$

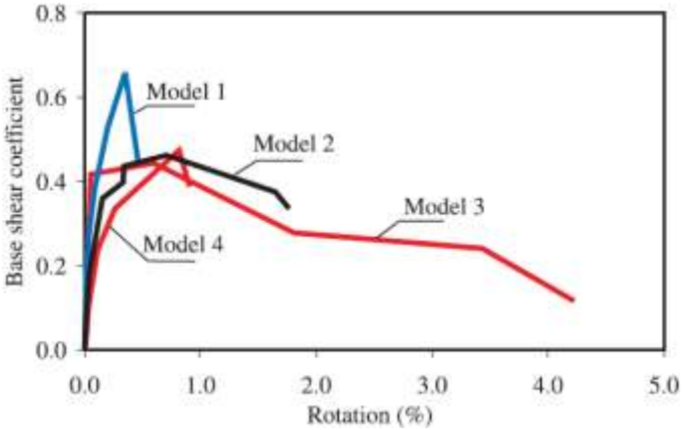
$$q = \frac{F_{el,max}}{F_{el}} = \frac{F_{el,max}}{F_y} \cdot \frac{F_y}{F_{el}} = q_0 \cdot \frac{F_y}{F_{el}} = q_0 \cdot OSR$$

Energy Dissipation Capacity (II)

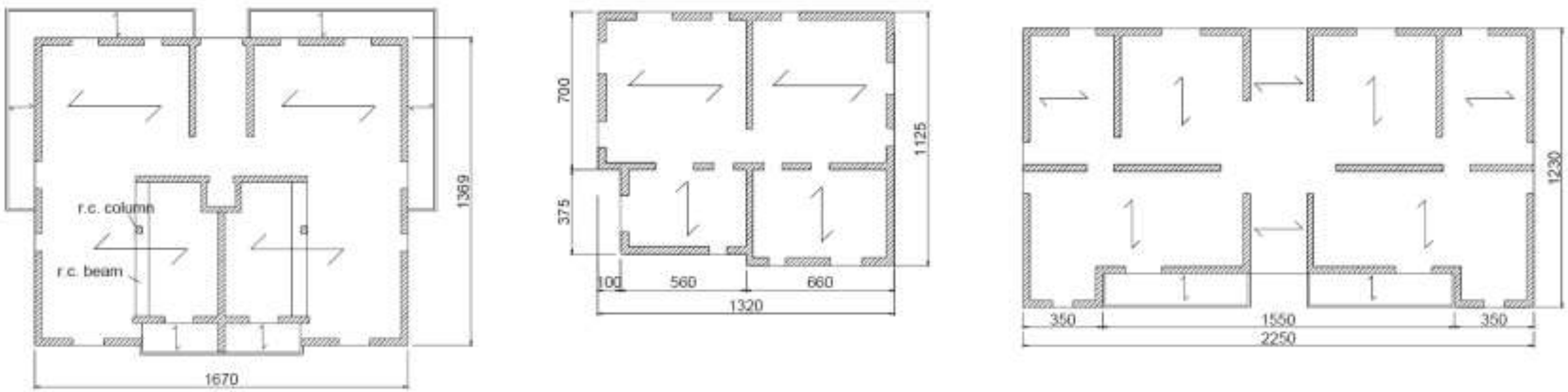


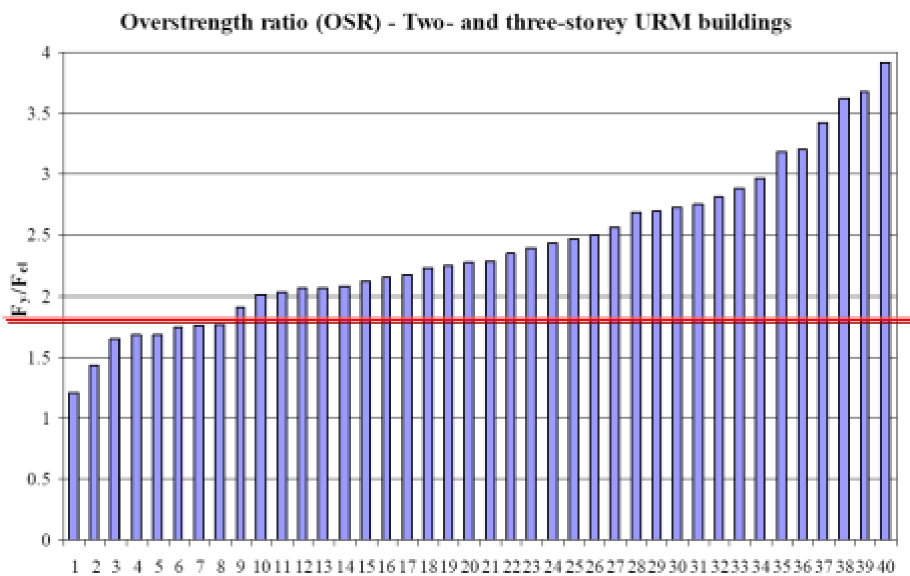






Energy Dissipation Capacity (III)





$q_{(0)} = 2.0$

OPCM 3431:

α_u / α_1 **(OSR) = 1.8**

$q = q_0 \times OSR = 3.6$

$$q = \frac{F_{el,\max}}{F_{el}} = \frac{F_{el,\max}}{F_y} \cdot \frac{F_y}{F_{el}} = q_0 \cdot \frac{F_y}{F_{el}} = q_0 \cdot OSR$$

Application

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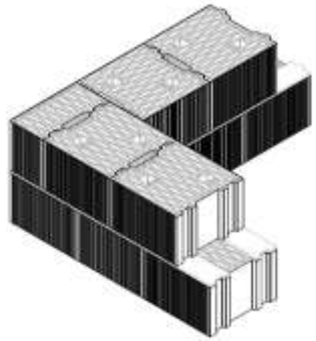
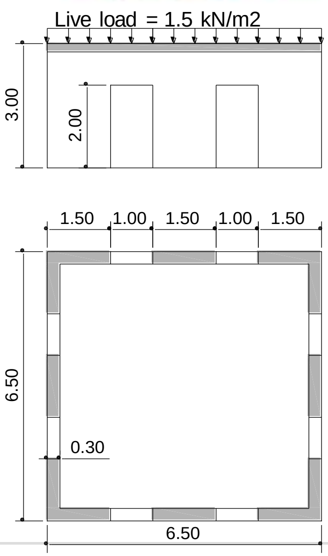
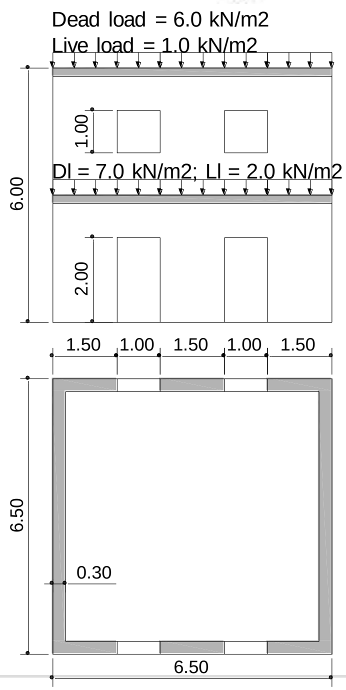
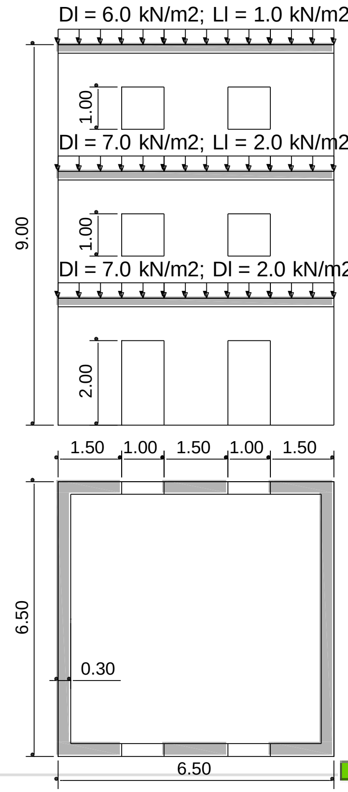
PARAMETRIC STUDY on the NR. of STOREYS (I)

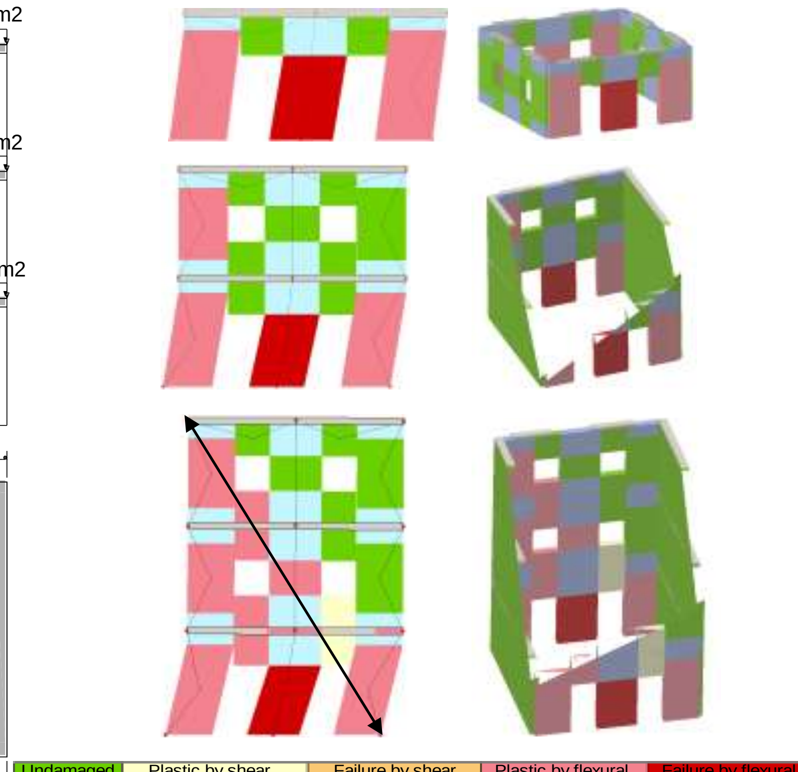


Specific weight , γ	17.0 kN/m3
Compressive characteristic strength , f_k	2.56 MPa
Sliding pure shear characteristic strength, f_{vk0}	0.15 MPa
Normal elasticity module, E	2560 MPa
Tangential elasticity module, G	1024 MPa

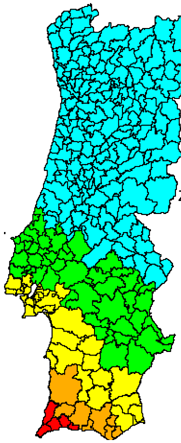




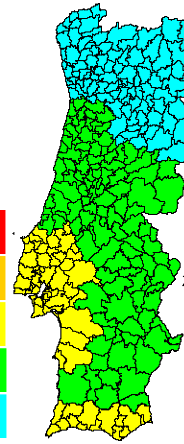




PARAMETRIC STUDY on the NR. of STOREYS (II)

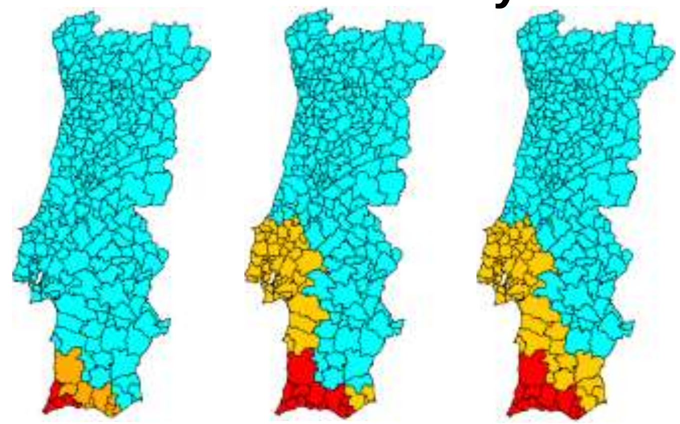


Zone	Soil	a_{gR} (m/s2)	S
1.1	A	2.50	1.00
	B		1.20
1.2	A	2.00	1.00
	B		1.20
1.3	A	1.50	1.00
	B		1.20
1.4	A	1.00	1.00
	B		1.30
1.5	A	0.50	1.00
	B		1.30

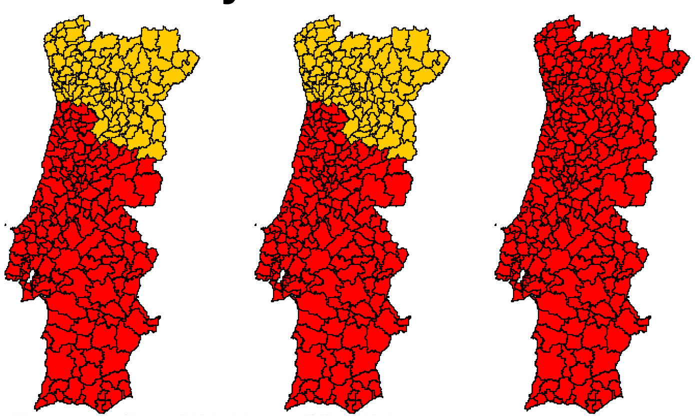


Zone	Soil	a_{gR} (m/s2)	S
2.3	A	1.70	1.00
	B		1.35
2.4	A	1.10	1.00
	B		1.33
2.5	A	0.80	1.00
	B		1.35

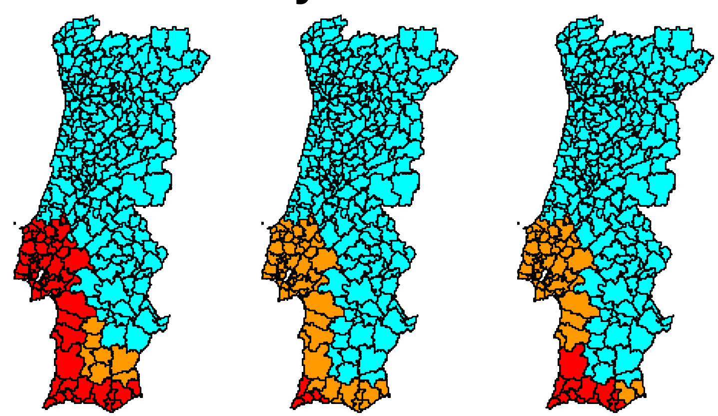
Pushover Analysis



Elastic Analysis ac. PT NA to EC8 (q=1.5)



Elastic Analysis ac. IT OPCM 3431



Unsafe Safe in soil type A Safe in soil types A and B

Masonry Structures Without Box Behavior

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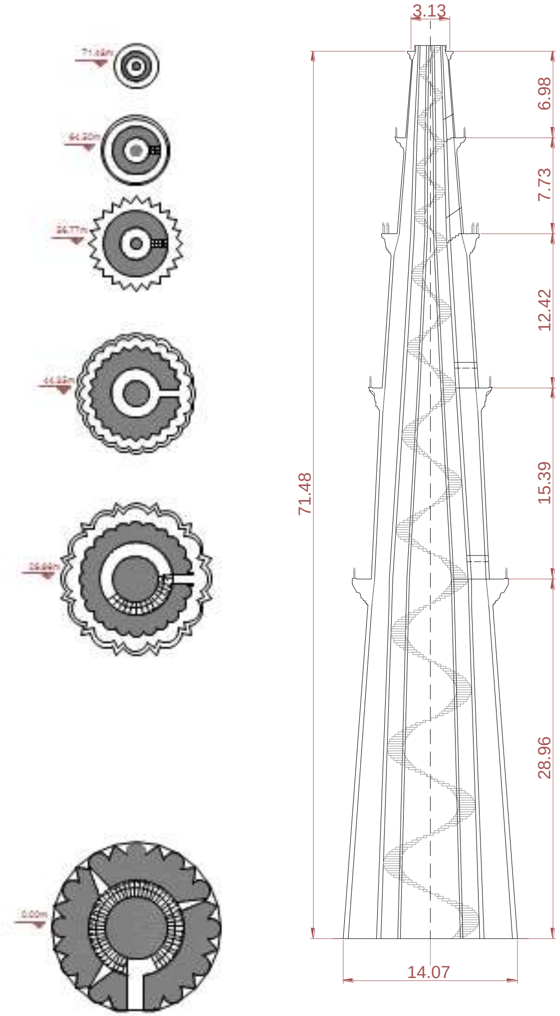
Recent Tests: Flexible Diaphragm

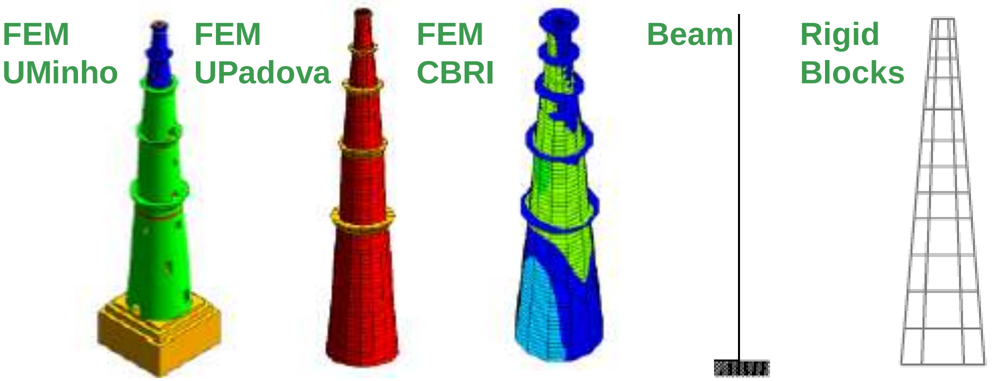
- "Gaioleiro"-type structure (late 19th century / early 20th century)
- Moderate damage for 100% of the design earthquake in Lisbon
- Light strengthening and collapse for 150% of the design earthquake in Lisbon

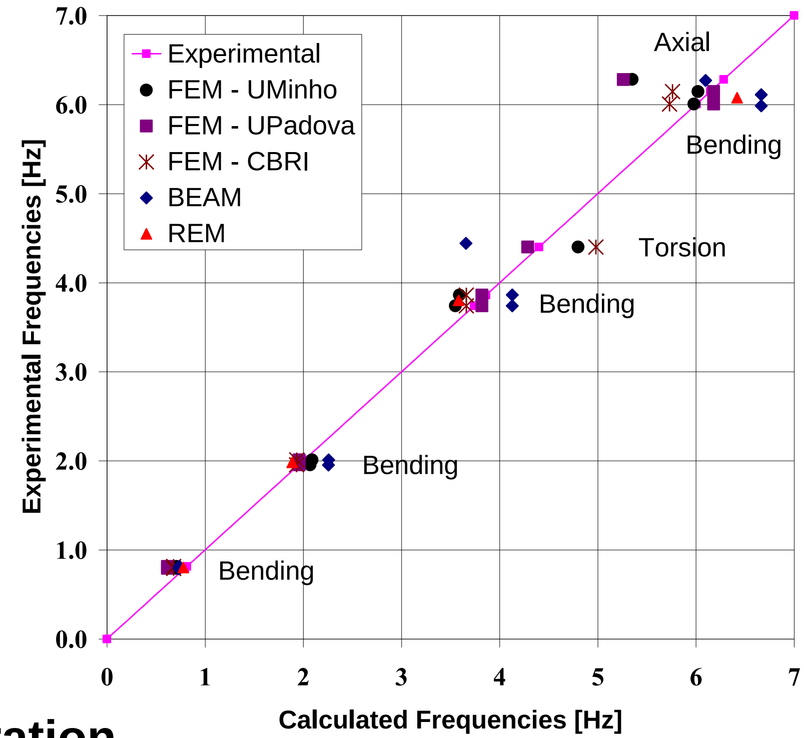
Qutb Minar



Location: New Delhi (India)
Material: Masonry
Total Height: 72.5 m
Crosss section: shell (3 leaves) + core (2 leaves)

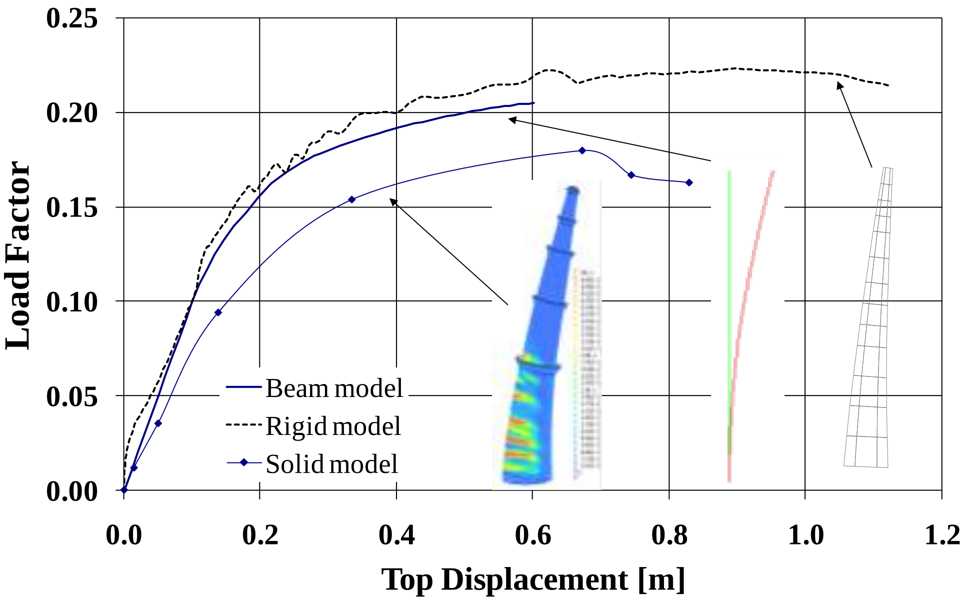
Numerical Modeling





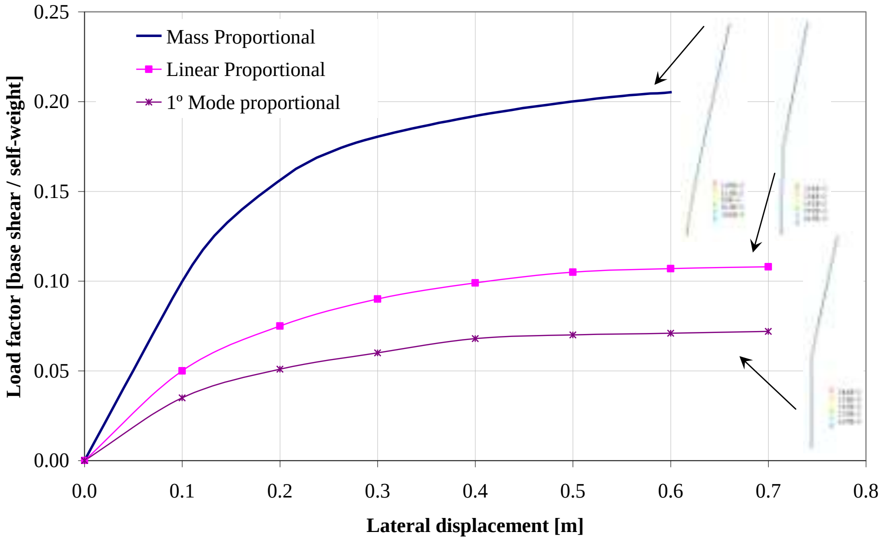
Calibration

Push-Over Analysis



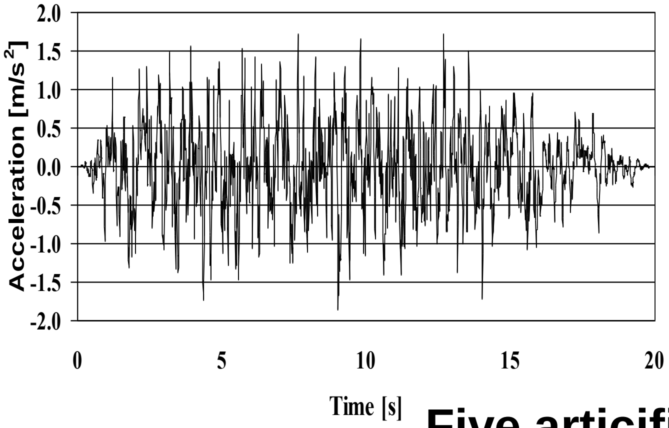
Uniform Mass Distribution

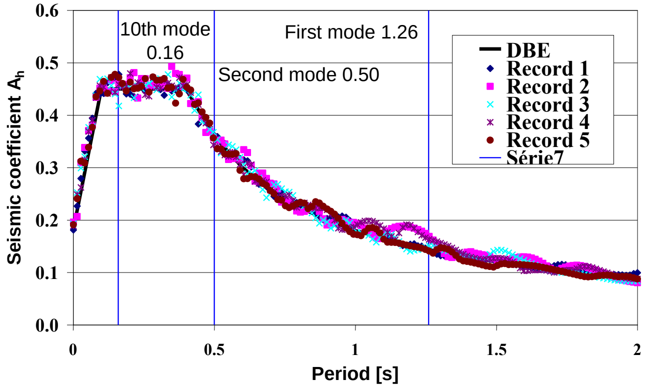
Collapse at the base



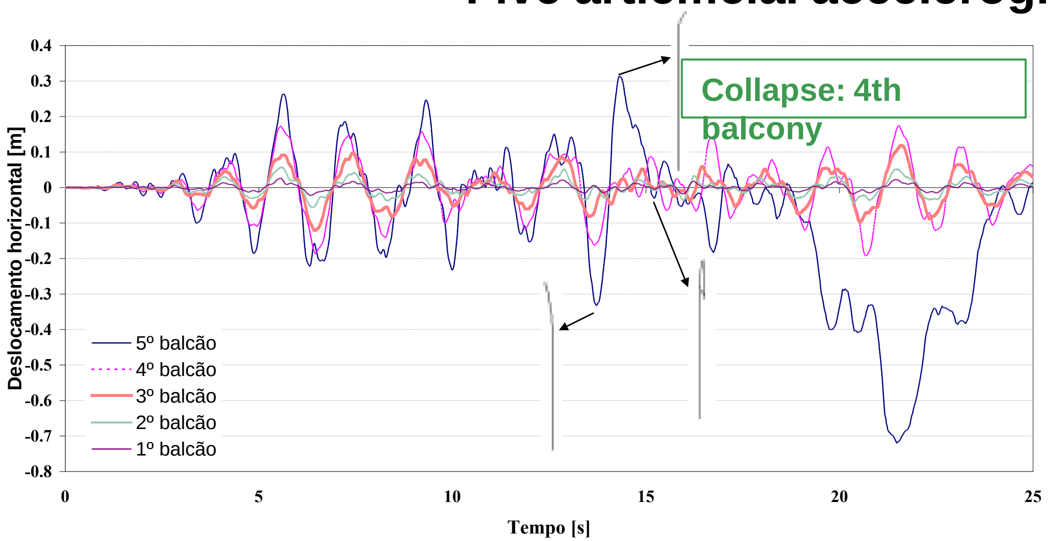
Other Mass Distributions

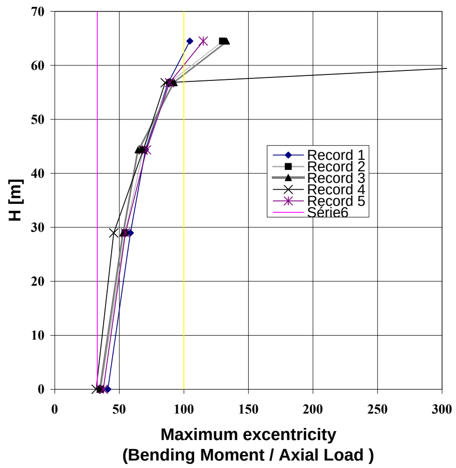
Time History Analysis





Five artificial accelerograms





REM

FEM – Collapse for 0.20*g*

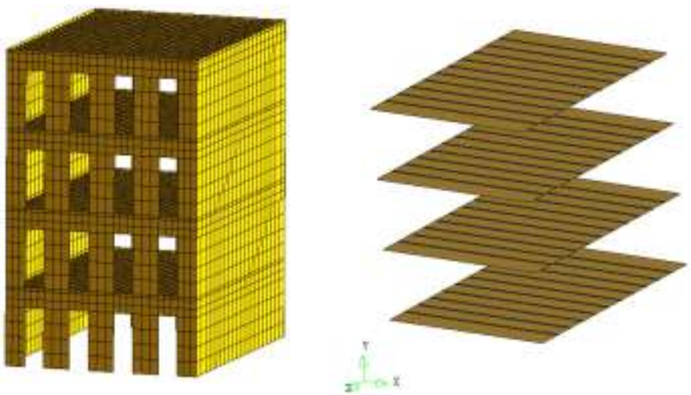
“Gaioleiro” Building

Location: Lisboa

Material: Masonry walls and timber pavements

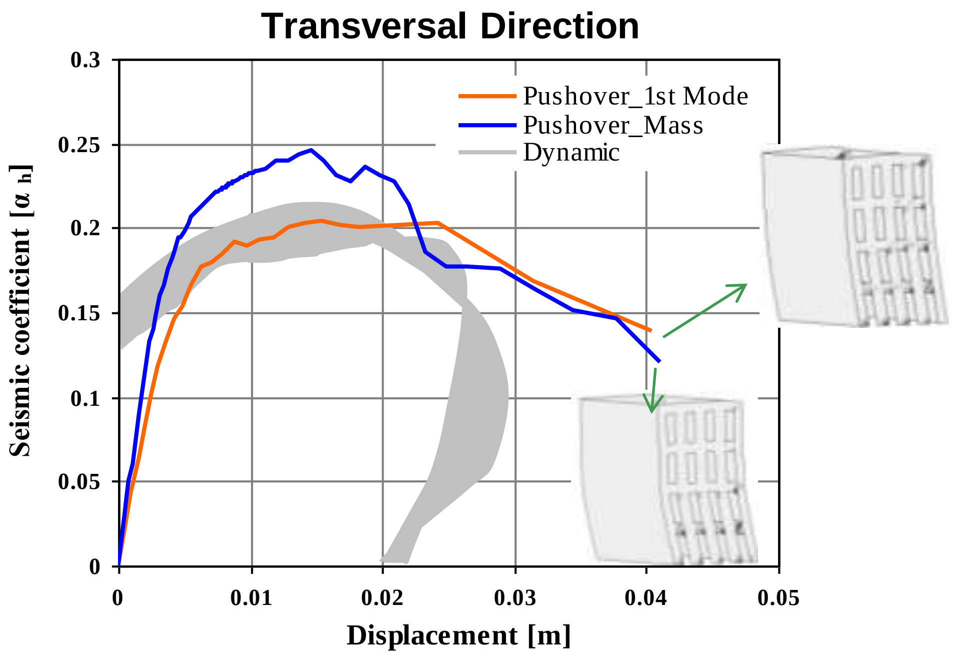
No. of storeys: 4 to 6

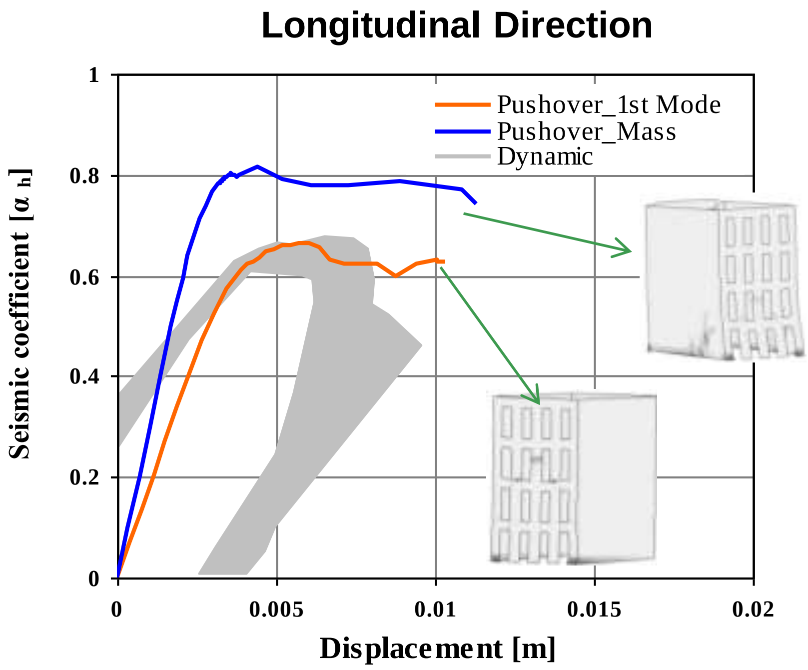
Numerical model



“Gaioleiro” Building

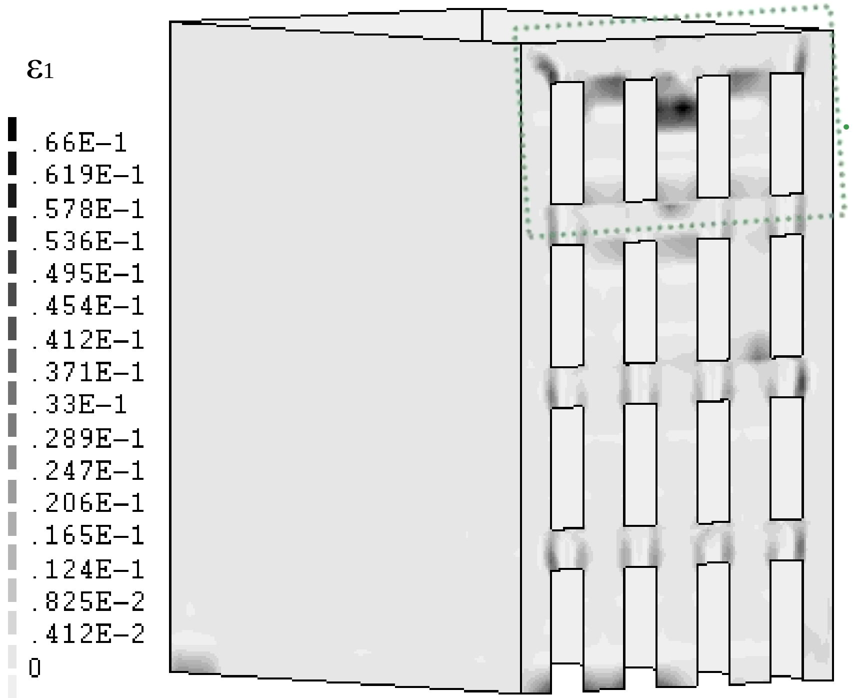
Pushover Analysis





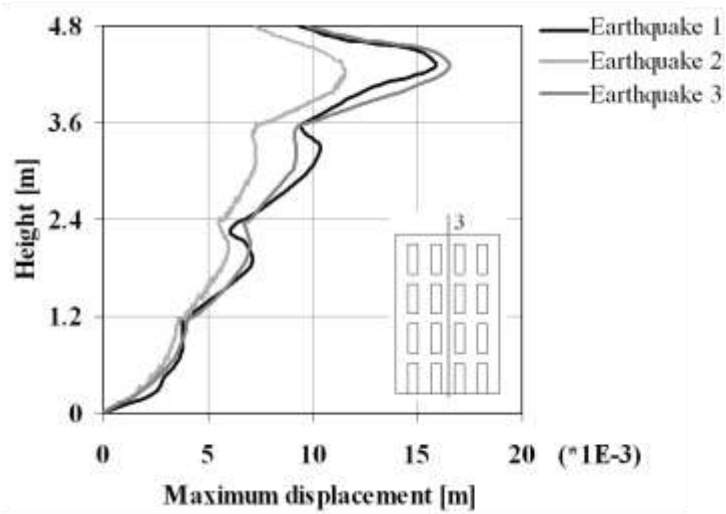
Time History Analysis

Numerical model



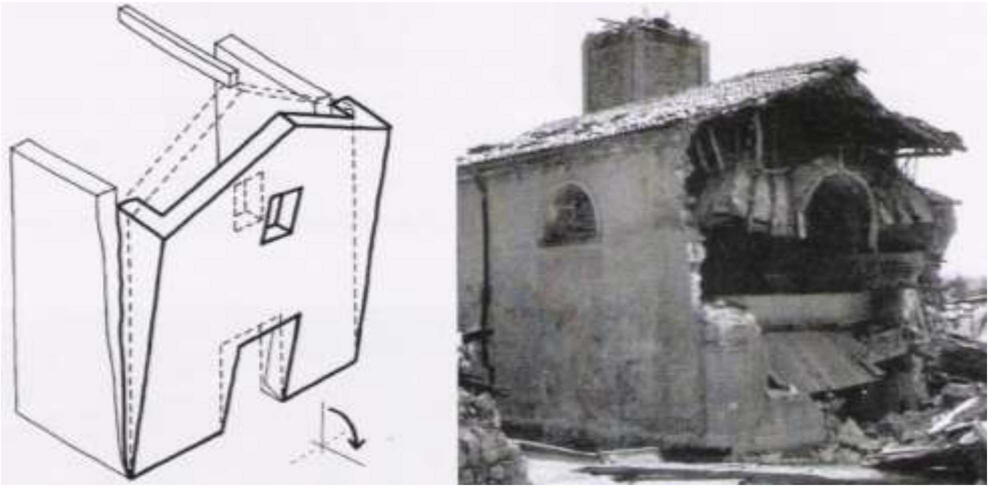
Principal strains (external surface)

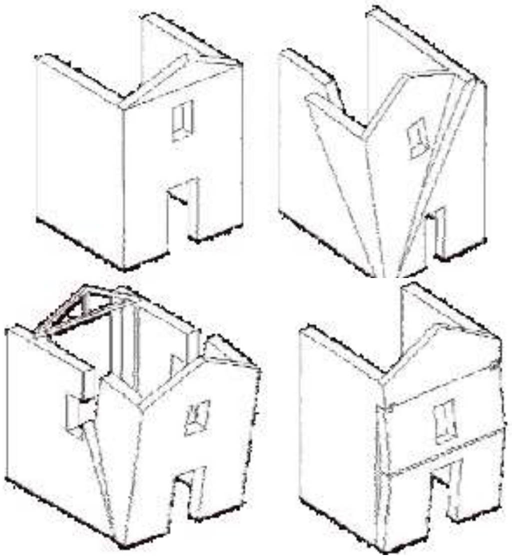
Experimental model



Design and Assessment = Macro-block analysis?

- Limit equilibrium analysis using the principle of virtual work is currently understood as the "best" analysis technique





Overturning

המועצה לשימור
אתרי מורשת בישראל

Conclusions

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Conclusions

- Design and assessment methods based on non-linear analysis should be used for masonry structures. Linear elastic analysis methods (application of “equivalent” static forces and modal superposition) are questionable
- Adequate models and commercial software, based on pushover analysis, are available for masonry with box behavior
- It was shown that pushover analyses do not simulate correctly the failure mode of masonry structures without box behavior, meaning that higher vibration modes have a significant contribution
- Pushover analysis proportional to the mass are probably the best solution is global structural analysis models are used
- For design purposes, particularly for strengthening design, macro-block limit analysis is probably the best analysis tool for practitioners
- More research needs to be done in the field of masonry structures without box behavior and earthquakes

Types of analysis: Linear static, linear dynamic and non linear static

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